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# Heat flow visualization for unsteady Casson fluid past a vertical slender hollow cylinder

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ABSTRACT

The conjugate heat transfer (CHT) effects have been studied on the heat function concept. The physical model consists of the Casson fluid flowing past a slender vertical cylinder. The inner wall of the hollow cylinder is maintained with identical temperature. The solutions of the governing equations which are coupled and non-linear are found by applying implicit scheme. The plots on the flow are depicted pictorially for all the controlling parameters. The steady-state time is extended for the Casson fluid parameter. The Bejan's heat flow visualization implies that the heat function contours are compact in the neighbourhood of leading edge at the wall of the cylinder having more temperature. The deviations of the heatlines from the hot wall go on reducing by escalating the values of all the controlling parameters. The Casson fluid plays a significant role at the hot wall when compared to the Newtonian fluid.

## 1. Introduction

The flow of viscous fluids subjected to unsteady natural convection has a broad range of applications. Sparrow and Gregg [1] analyzed free convective flow through a uniformly heated cylinder. Lee et al. [2] studied the similar problem with power-law fluid along the thin vertical cylinder. Such studies on the rheological fluids are enviable for their growing importance in the different fields of science and engineering. In the previous studies it was noticed that the heat transfer between the fluid and the cylindrical wall was ignored. Further it was also assumed that the cylindrical rigid wall is skinny. For the industrial applications it is required to understand the physical characteristics of transfer of heat at the interface occurring between the solid and the fluid. Therefore, the convection in the fluid and the conduction in the solid interface should be determined at the same time. Following the above assumption, the transfer of heat is named as the CHT process. This kind of CHT problems has wide-range of real life applications, such as, energy preservation in buildings, cold storage fittings, cryogenic uses, medical and space technology. Many researchers have widely studied this type of CHT research problems [3–8], with the help of mathematical modeling for different flow geometries. Such studies on non-Newtonian fluids are enviable for their growing importance in the real world problems. But a very few non-Newtonian fluid problems in fluid dynamics received attention, because of their unique challenge to engineers, physicists, and mathematicians. The Soret and Dufour effects were examined by Rani and Reddy [9] in a vertical cylinder for the couplestress fluid. MHD flow for the power law fluid flowing over a flat plate was discussed by Hirschhorn et al. [10]. MHD flow past a permeable surface with heat source and sink was analyzed by Khedr et al. [11] using micropolar fluid.

Honey, jelly, concentrated juices of the fruits, soup, blood of the human being, tomato sauce, etc. in the classification of polar fluid theories are under the category of Casson fluid, and this fluid fits the data obtained from the rheology in a better way when compared with the plastic models with viscoelastic in nature. This fluid defines as a shear dilution liquid with indefinite viscosity in the absence of shear, and no flow occurs below the yield stress and an infinite rate of shear with zero viscosity [12]. Some researchers [13,14] studied the heat transfer for Casson fluid using suitable transformations. Abbas et al. [15] and Mustafa et al. [16] analyzed similar flow problem of transfer of heat past a stretching surface/sheet under different conditions. Flow properties for the Casson fluid flowing past a wedge with symmetry was investigated by Mukhopadhyay et al. [17]. Time-dependent Casson fluid flowing over a plate and a cone under the effects of radiation in the presence of chemical reaction was studied by Mythili and Sivaraj [18]. Das et al. [19] studied the time-dependent electrically conducting

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**Nomenclature**

| | |
|---|---|
| $\overline{C_f}$ | coefficient of average momentum transport |
| $c_p$ | specific heat ($m^2 \cdot s^{-2} \cdot K^{-1}$) |
| $e_{ij}$ | deformation rate ($s^{-1}$) |
| $Gr$ | Grashof number |
| $g$ | gravity ($m \cdot s^{-2}$) |
| $k_f$ | thermal conductivity ($kg \cdot m \cdot s^{-3} \cdot K^{-1}$) |
| $k_s$ | solid thermal conductivity ($kg \cdot m \cdot s^{-3} \cdot K^{-1}$) |
| $L$ | length of hollow cylinder (m) |
| $\overline{Nu}$ | dimensionless average heat transfer coefficient |
| $P$ | CHT parameter |
| $Pr$ | Prandtl number |
| $p_y$ | yield stress ($kg \cdot m^{-1} \cdot s^{-2}$) |
| $R$ | coordinate along radial direction without dimension |
| $r$ | coordinate along radial direction (m) |
| $r_0$ | radius for outer cylinder (m) |
| $r_i$ | radius for inner cylinder (m) |
| $T'$, $T'_0$ | temperature of the fluid and inner cylinder, respectively (K) |
| $T'_\infty$ | dimensional environment temperature (K) |
| $T'_w$ | dimensional fluid–solid interface temperature (K) |
| $t$ | time without dimension |
| $t'$ | time (s) |
| $U$, $V$ | velocity components in polar coordinate system without dimension |
| $u$, $v$ | the components of velocity in polar coordinate system ($m \cdot s^{-1}$) |
| $X$ | coordinate in axial direction without dimension |
| $x$ | axial coordinate (m) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity ($m^2 \cdot s^{-1}$) |
| $\beta$ | Casson fluid parameter |
| $\beta_T$ | thermal expansion coefficient ($K^{-1}$) |
| $\theta$ | temperature without dimension |
| $\mu$ | fluid viscosity ($kg \cdot m^{-1} \cdot s^{-1}$) |
| $\mu_B$ | plastic dynamic viscosity ($kg \cdot m^{-1} \cdot s^{-1}$) |
| $\nu$ | viscosity in kinematic form ($m^2 \cdot s^{-1}$) |
| $\pi$ | deformation rate product ($e_{ij}e_{ij}$) ($s^{-2}$) |
| $\pi_c$ | critical value of deformation rate product ($s^{-2}$) |
| $\rho$ | density ($kg \cdot m^{-3}$) |
| $\tau_{ij}$ | stress tensor ($kg \cdot m^{-1} \cdot s^{-2}$) |
| $\psi$ | stream function without dimension |
| $\Omega$ | dimensionless heat function |
| $\Omega'$ | dimensional heat function ($kg \cdot m^2 \cdot s^{-3}$) |

*Subscripts*

| | |
|---|---|
| $a$, $b$ | grid levels in ($X$, $R$) coordinate system |
| $w$ | wall conditions |
| $\infty$ | ambient conditions |

*Superscripts*

| | |
|---|---|
| $n$ | time level |

Casson fluid past a radiative plate by considering chemical reaction effect. Nadeem et al. [20] also discussed about the electrically conducting Casson fluid. The mass transfer effects were observed by Raju et al. [21] using the Casson fluid over the stretching sheet having pores.

Thus far relatively scant attention has been directed towards mathematical modeling of Casson fluid convection from a slender vertical cylinder. The present work is therefore focused on free convection in Casson viscoplastic boundary layer flow from over a uniformly heated slender vertical cylinder. The temperature of the inner cylindrical wall is maintained uniformly and is greater than the surrounding fluid temperature. The outer cylinder wall temperature is resolved by the conjugate solution of the time-independent state thermal energy equation of the solid and fluid flow. The transitory effects of the Casson fluid flow are studied for the momentum and heat transport coefficients for different control parameters and compared with the Newtonian fluid flow. The results obtained by the implicit scheme are corroborated with the available results in the literature. It should be noted that the obtained results can be used for design of automatic cooking machines and the design of internal engine parts in mechanical engineering [22]. Other applications arise in petroleum production, multiphase mixtures, pharmaceutical formulations, coal in water, paints, lubricants, jams, sewage, soup, blood. Moreover, Casson fluid flow in cylindrical geometry has important applications in blood flow [23,24].

Usually, the fluid dynamics problems are analyzed only with the help of streamlines and isotherms. In a given domain isotherms will contribute to illustrate the temperature distribution. But using them a visualization of the direction and heat transfer intensity is not possible to analyze. Especially in convective problems due to the convection effect the direction of heat flux is not normal to temperature contours. In such type of challenges, the heatlines are the boon to visualize and investigate the intensity of heat transfer. These will give well-defined corridors for the energy transfer to occur from hot to cold walls. Kimura and Bejan [25] and Bejan [26] initiated the heatline concept of flow visualization. For cylindrical enclosures, Aggarwal and Manhapra [27,28] employed heatlines for the unsteady natural convective heat transfer process. Rani and Reddy [29] studied the heatlines for couple stress fluid past a slender vertical cylinder. It is worth noting that the concept of couple stresses arises due to the way in which the mechanical interactions in the fluid medium are modeled. This theory describes the flow behavior of fluids containing a substructure such as lubricants with polymer additives, liquid crystals and animal blood [30,31]. Many studies have been made on the hydrodynamic lubrication of squeeze film flows considering the lubricant as a couple stress fluid and the studies revealed that the couple stress fluid increases the load carrying capacity of the journal bearing [32,33]. Later Rani et al. [34] studied the similar problem like in [29] and investigated the massline visualization concept. Using the same idea, recently Das and Basak [35] analyzed the rate of heat transfer at different zones within enclosures involving discrete heaters. Use of heatlines approach for convection problem is shown in [36–41]. Till date, very less attention has been given in the literature for the heatlines concept. Also, for this problem first time, an endeavor is made to visualize the flow behavior using the heat function concept.

The organization of this research article is as follows: Section 2 presents the mathematical formulation and its non-dimensionalization for a Casson fluid flow from a slender vertical cylinder with inner wall reserved at a uniform surface temperature. Next section deals with the grid generation and numerical method to solve the flow-field equations. In the results and discussion section 4, the transient two-dimensional flow-field profiles, average wall and heat transfer rates are analyzed. Also, the heat function has been derived and non-dimensionalized based on Nusselt number. The comparison between the flow due to the Casson and Newtonian fluids is shown and analyzed. Finally, in section 5 the concluding remarks are made.