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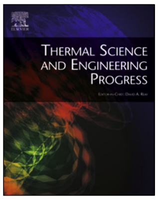

# Numerical modelling of Fanno flows in micro channels: a quasi-static application to air vents for plastic moulding

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ABSTRACT

The flow in micro air vents, as those used in the plastic moulding industry, can be assimilated to a Fanno flow. Even though Fanno theory is well-established, its outcome strongly depends on the correct assessment of the friction factor term. Yet, friction factor evaluation is not straightforward since traditional fluid-dynamics formulas fail to account for the compressibility effects and are only valid when low Mach numbers are attained, which is often not the case. A numerical model implementing the non-isoentropic compressible Fanno flow theory is presented. A friction factor correlation is proposed, based on the results of a large set of CFD simulations used for calibrating the numerical model. The model is then used for characterizing micro air vent geometries under different operating stagnation pressures and temperatures. The vents characterization is finally employed for the quasi-static assessment of the mass flow rate through the vents, and of the air pressure in the mould during a typical moulding process.

## 1. Introduction

In recent years the interest in micro-scale fluid flow and heat transfer has grown significantly. This is mainly due to the technological advances that make the constant quest for miniaturization possible. The range of applications where micro-scale channels are employed, or where micro-scale fluid flow is of interest at any level, is large, and probably finds its core in the fields of micro-electro-mechanical systems and of refrigeration. Not being limited to that, applications span from aerospace engineering to advanced energy systems, bioengineering, vacuum or pressurized systems, and carbon nanotubes, to cite a few.

Micro-flows are characterized by the fact that surface friction becomes dominant over the inertial effects. Thus, when it comes to gas flow, compressibility cannot be neglected. The classical one-dimensional theory of compressible flows, as found in any thermodynamic textbook [1], considers only compressible isoentropic flows through nozzles. More elaborated 1D theories, have been derived during the first half of the last century and their theoretical basis are well-established: these are known as Fanno, and Rayleigh flows. In Fanno flow the effects of friction are addressed, while the flow is assumed adiabatic. On the contrary, Rayleigh flow includes heat transfer effects, while neglecting friction. Both the models assume a constant cross-section channel, crossed by a stationary flow of an ideal gas.

Fanno and Rayleigh flows are based on a set of three first-order differential equations (conservation of mass, momentum and energy), plus the equation of state for gases. As such, three independent boundary or initial conditions are required to univocally solve the flow [2]. These are, commonly, the upstream stagnation temperature and pressure, and the downstream stagnation pressure. From conservation, these models derive a set of equations linking the local variation of the different properties (such as temperature, pressure, density, Mach number, and so on) to the local Mach number and friction factor, or heat flux. A direct solution of the equations, thus, is not viable as long as the Mach number at the channel entrance, or at some location along the channel, is not known. For this reason, Fanno and Rayleigh flow equations are generally solved iteratively, until the inlet Mach number that meets the boundary conditions imposed is found.

Different approaches and generalizations to the solution of the Fanno and the Rayleigh flows have been tried in the literature under different assumptions. For instance, in [3] an extension is given allowing the problem to be solved also for the case of channels with variable cross-section. However, this is made possible only if an additional function relating the change in pressure to the change in area is hypothesized.

Considering the Fanno flow alone, in [4], a direct approximate method of solution is proposed and discussed. The need of an iterative procedure is avoided if a constant value for the friction factor in the channel is assumed *a priori*, yet such an assumption is far from true unless the Mach number is low. The same idea is developed in [5], where a number of considerations on how to

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## Nomenclature

| | |
|---|---|
| $a$ | JANAF coefficients |
| $A$ | channel cross-section area |
| $c$ | speed of sound |
| $c_p$ | heat capacity at constant pressure |
| $C_f$ | friction correction factor |
| $C_s$ | Sutherland coefficient |
| $c_v$ | heat capacity at constant volume |
| $D$ | channel diameter (circular cross-section) |
| $D_h$ | channel hydraulic diameter |
| $f$ | Darcy friction factor |
| $F_v$ | viscous forces resultant |
| Fa | Fanno number |
| $k$ | concentrated pressure loss |
| $l$ | distributed pressure loss |
| $L$ | channel length |
| $L^*$ | length required to reach the choked flow |
| $h$ | specific enthalpy |
| $H$ | channel height (parallel-plate) |
| Kn | Knudsen number |
| $m$ | mass |
| $\dot{m}$ | mass flow rate |
| Ma | Mach number |
| $p$ | pressure |
| $P$ | channel cross-section perimeter |
| Po | Poiseuille number |
| $R$ | specific gas constant |
| Re | Reynolds number |
| $t$ | time |
| $T$ | temperature |
| $T_s$ | Sutherland temperature |
| $U$ | velocity |
| $v$ | specific volume |
| $V$ | volume |
| $\dot{V}$ | volume flow rate |
| $x$ | longitudinal coordinate |

*Greek Symbols*

| | |
|---|---|
| $\alpha$ | channel aspect ratio ($= L/D_h$) |
| $\beta$ | channel cross-section aspect ratio ($< 1$) |
| $\gamma$ | heat capacity ratio |
| $\Delta$ | difference |
| $\mu$ | dynamic viscosity |
| $\rho$ | density |
| $\tau_w$ | wall shear stress |

*Subscripts*

| | |
|---|---|
| 0 | upstream stagnation section |
| 1 | downstream stagnation section |
| $a$ | relative to the generic section $a$ |
| $b$ | relative to the generic section $b$ |
| $cmp$ | compressible |
| $d$ | dynamic |
| $i$ | relative to the channel inlet section |
| $inc$ | incompressible |
| $lam$ | laminar |
| $o$ | relative to the channel outlet section |
| $r$ | stagnation to local value ratio |
| $t$ | total |
| $trb$ | turbulent |

*Superscripts*

| | |
|---|---|
| $[i]$ | relative to the $i$-th node or segment |
| $(j)$ | relative to the $j$-th time step |

properly guess the friction factor are given. Another attempt to avoid the iterative approach is given in [6], where a method for the direct evaluation of the inlet Mach number is given, but only for the case of choked non-isoentropic flow. An interesting, closed form mass flow rate equation is given in [2] under the assumption of subsonic Fanno flow in constant circular cross-section pipes.

Several works also extended the Fanno flow theory in a multiphase sense. This research is mainly pushed by the needs of the refrigeration and the nuclear industry, where two-phase Fanno flow finds some of its more common applications. A key aspect for the closure of the resulting system of equations is the definition of the slip ratio, that is, the ratio between the velocities of the different phases, as discussed in [7]. Another extension is the one considering real gas effects into the Fanno model. This has been studied isoentropically in [8], and non-isoentropically in [9]. The latter work focuses on the dense gas condition, a particular state where choking cannot occur because of the rapid change of the fluid properties with pressure and temperature.

A number of varied applications of Fanno flows is also found in the literature. In [10] a Fanno-based model for circular porous air bearings is developed. The goal is to provide a simple yet reliable design tool that could substitute more complicated 3D CFD simulations. In [11] the Fanno model is applied to solve pneumatic pipelines in series. A failure analysis of a pipeline is given in [12], where gas leakages through cracks are modelled as Fanno flows in micro-channels. Still following the Fanno theory, the work in [13] deals with an application to supercritical fluid flows, while [14] investigates ejector refrigeration systems. An interesting, yet more academic, application is given in [15], where it is shown how in a pipe characterized by a series of sudden expansions, multiple choked compressible flow can be attained. When this occurs, the upstream critical section is found to be the one limiting the mass flow rate in the whole system.

Despite the well-established models developed by Fanno and Rayleigh, the study of compressible flows in micro-channels is a particularly challenging topic for a number of reasons. First of all, the extremely small dimensions of the channels, and the high velocities reached as the flow approaches choking, make the experimental analysis very complicated, most of the time limiting its reliability. As reminded in [16], for instance, this is particularly true for temperature measurements, which yet provide important information for the assessment of the friction factor, and/or the heat flux. These quantities stand at the basis of the whole analytical approach, and are fundamental for its accuracy. Secondly, most of the times the analytical fluid flow and heat transfer characteristics deviate unexpectedly from experience as the Mach and the Reynolds numbers grow. Many applications in the literature, in fact, avoid that region.

As a consequence of a wide collection of questionable results, most of the time in contradiction to each other, and of the difficulties met in the setup of a reliable analytical model, the scientific community started to question itself on the applicability of the traditional formulas of fluid dynamics to micro-flows. The debate has been long and controversial, and a widely accepted solution has begun to unravel only in recent years.

For sure Navier–Stokes equation and the traditional fluid dynamics formulas fall when the continuum hypothesis is no longer applicable. This is the case when the Knudsen number,