Thermal Science and Engineering Progress 2 (2017) 64–70

Contents lists available at ScienceDirect

Thermal Science and Engineering Progress

journal homepage: www.elsevier.com/locate/tsep

Thermal dispersion effects on forced convection in a porous-saturated pipe

K. Hooman [a,*], J. Li [b,c], M. Dahari [d]

[a] School of Mechanical and Mining Engineering, The University of Queensland, Qld 4072, Australia
[b] Key Laboratory of Efficient Utilization of Low and Medium Grade Energy, Ministry of Education, Tianjin University, Tianjin 300072, China
[c] State Key Laboratory of Coal Resources and Safe Mining, China University of Mining & Technology, Xuzhou 221116, Jiangsu Province, China
[d] Faculty of Engineering, University of Malaya, Kuala Lumpur, Malaysia

ARTICLE INFO

Article history:
Received 16 February 2017
Received in revised form 15 April 2017
Accepted 17 April 2017

Keywords:
Thermal dispersion
Asymptotic technique
Numerical
Cylindrical tube
Porous media

ABSTRACT

Thermal dispersion effects on fully developed forced convection inside a porous-saturated pipe are investigated. The pipe wall is assumed to be kept at a uniform and constant heat flux. Having the fully developed velocity field furnished by an arbitrary power series function, the energy equation is solved using asymptotic techniques for the limiting case when thermal conductivity, as a result of thermal dispersion, weakly changes with the Péclet number. A numerical solution, valid for the entire range of thermal dispersion conductivity, is also presented. This latter solution is presented to check the accuracy of the former. The two solutions are then cross-validated in the limits. Besides, results are found to be in good agreement with those previously reported in the literature.

1. Introduction

Because of its relevance to a variety of engineering applications including geothermal systems, underground fire control, coal and grain storage, solid matrix heat exchangers, and energy recovery in high temperature furnaces, convection in porous media is a well-developed field of investigation [1–3]. Porous heat exchangers were investigated for their possible applications in solar thermal plants [4], cooling towers [5], electronic cooling [6], exhaust gas recirculation for diesel engines [7] and thermal storage systems [8]. The effects of thermal dispersion on convection in porous media have been analyzed in details as surveyed in [9]. Closed form solutions, to fully developed thermal energy equation, can be obtained for the case of Darcy flow. However, with the boundary, inertia and convective term effects included in the fully developed momentum transfer equation in porous media, no analytical solution has been reported in the literature. Furthermore, thermal analysis of the problem has to rely on a prescribed velocity field. This is because thermal dispersion conductivity is a function of the (volume-averaged) fluid velocity which is not uniformly distributed over the duct cross-section. In fact, a core region is observed away from the walls while the velocity sharply changes near the walls [10]. Hence, analytical solution to the temperature distribution can be obtained as a combination of log, hyperbolic and polygarithm functions with imaginary arguments [9] which are too complex to be useful in engineering applications where a quick assessment of heat transfer through porous media is of primary interest. Numerical simulations have been exclusively used in the literature for such non-Darcy flow problems [11–14]. Hunt and Tien [15] experimentally studied non-Darcian forced convection flow and heat transfer in high-porosity fibrous media. A model was put forward for thermal dispersion and the adequacy of a homogeneous energy equation to model the transport was ascertained.

Thermal dispersion tensors were calculated within an infinite porous medium formed by a spatially periodic array of longitudinally-displaced elliptic rods by Pedras and de Lemos [16]. The authors applied a low Reynolds k–ε closure for turbulence and investigated the effects of solid-fluid thermal conductivity ratio using a unit-cell geometry in conjunction with periodic boundary conditions for mass, momentum and energy equations. Cell-integrated results indicated that compared to the longitudinal dispersion coefficient, the transversal counterpart is more sensitive to porosity, the applied boundary condition type, medium morphology and solid–fluid conductivity.

Two methods of volume average and multiple scale expansion were undertaken to model the thermal dispersion in a rigid

* Corresponding author.
E-mail address: k.hooman@uq.edu.au (K. Hooman).

http://dx.doi.org/10.1016/j.tsep.2017.04.005
2451-9049/© 2017 Published by Elsevier Ltd.

Nomenclature

a,b,c	coefficients of the series [–]
B	dimensionless constant [–]
C	perturbation parameter [–]
C_d	thermal dispersion coefficient [–]
C_F	form drag coefficient [–]
c_P	specific heat at constant pressure [j/kg K]
D_1	integration constant [–]
Da	Darcy number [–]
F	dimensionless form drag coefficient [–]
G	applied pressure gradient [Pa/m]
I	Bessel function [–]
k	thermal conductivity [W/m.K]
K	permeability [m^2]
M	μ_{eff}/μ [–]
Nu	Nusselt number [–]
Pe	Péclet number, Pe = RePr [–]
Pr	Prandtl number, $\mathrm{Pr} = \frac{\mu c_p}{k_f}$ [–]
R	pipe radius [m]
Re	Reynolds number, $\mathrm{Re} = \frac{\rho U \sqrt{K}}{\mu}$ [–]
q''	wall heat flux [W/m^2]
s	$(M\mathrm{Da})^{-1/2}$ [–]
T^*	temperature [K]
T_m	bulk mean temperature [K]
T_w	downstream wall temperature [K]
u	$\mu u*/GR^2$ [–]
u^*	filtration velocity [m/s]
$\hat{u}$	u^*/U [–]
U	mean velocity [m/s]
(x^*, r^*)	coordinate system [m]
r	r^*/R [-]

Greek symbols

θ	$(T^* - T_w)/(T_m - T_w)$ [–]
ϕ	*porosity* [–]
κ	thermal conductivity ratio $\frac{k_f}{k_e}$
μ	dynamic viscosity [Pa.s]
μ_{eff}	effective dynamic viscosity [Pa.s]
ψ	modified dimensionless temperature [-]
ρ	fluid density [kg/m^3]

Subscripts

(0), (1)	term sequence in asymptotic expansion
d	dispersion
e	effective
f	fluid
n	term sequence in power series expansion
new, old	velocity values in successive (old/previous and new/current) iterations
s	solid

homogeneous porous medium described by a periodic model in [17]. The theoretical longitudinal thermal dispersion coefficient for a stratified system was found to be in good agreement with those obtained through the use of a random walk method.

Cheng [18] investigated fully-developed flow in a rectangular and an annular packed bed using Van Driest's mixing length theory. The predicted heat transfer features were contrasted with the experimental data to obtain the constants in the mixing length theory as well as those in the expression for the transverse thermal dispersion.

Ozgumus and Mobedi [19] numerically investigated the effects of pore to throat size ratio on thermal dispersion of periodic porous media consisting of inline array of rectangular rods. The difference between macroscopic and microscopic values of temperature and velocity are computed numerically so that the thermal dispersion coefficients of the porous media can be determined. It was reported that for Re > 10, higher Reynolds number and porosity values increase both the transverse and longitudinal thermal dispersion coefficients. Interestingly, an optimum value for pore to throat size ratio was reported maximizing the longitudinal thermal dispersion coefficient.

Metzeger et al. [20] have evaluated the thermal dispersion coefficients for water flow through a packed bed of glass spheres by placing thermocouples in the downstream neighborhood of a line heat source to measure the temperature response to a step heat input. Monte Carlo simulations of measurements was performed to quantify the errors. Interestingly, it was reported that the assumption of the one-temperature model is reasonable even in the case of local thermal non-equilibrium.

Ozgumus et al. [21] reviewed the experimental studies conducted to determine the effective thermal conductivity of one class of porous media begin packed beds. The authors categorized the experimental works into three groups: (1) heating/cooling of the lateral boundaries, (2) heat addition at the channel inlet/outlet, (3) internal heat generation. Experimental details, methods, obtained results, and suggested correlations for the determination of the effective thermal conductivity were presented.

In order to by-pass the difficulty in the analysis of this problem, an asymptotic solution is presented here based on suggestions and simplifications discussed in [22]. These make our asymptotic solution valid for a range of thermal dispersion conductivity values. In forthcoming sections, the analysis of the problem is discussed along with numerical solutions for a wide range of thermal dispersion conductivity values. Comparison between the two solutions, sets the range of validity of the theoretical results obtained based on the asymptotic techniques.

2. Analysis

For the steady-state fully developed flow, we have unidirectional flow in the x^*-direction inside a porous-saturated pipe with impermeable wall at $r^* = R$, as illustrated in Fig. 1.

For $x^* > 0$, the (downstream) heat flux at the tube wall is held constant at the value q". The momentum Eq. [9] is

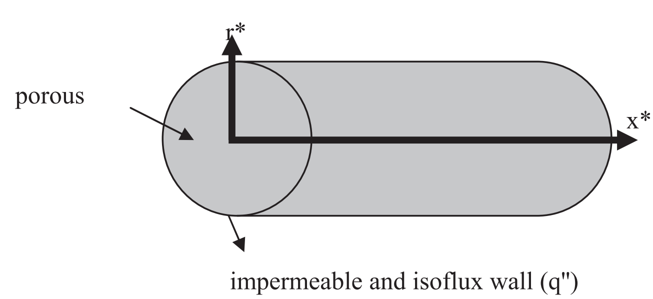

Fig. 1. Definition sketch.