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Full Length Article

Ternary photonic crystal with left-handed material layer for refractometric application

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ABSTRACT

A ternary photonic crystal with left-handed material (LHM) layer is examined for refractometric applications. One of the layers is assumed to be air and treated as an analyte. The transmittance from the ternary photonic crystal is studied in details and the wavelength shift due to the change in the refractive index of the analyte is investigated. The transmittance is investigated with the parameters of the LHM. It is found that the wavelength shift can be significantly enhanced with the decrease of both real part of the LHM permittivity and thickness.

1. Introduction

Many researchers were attracted to photonic crystals in recent years due to possible applications in optoelectronics [1–4]. This multilayer structure led to the construction of photonic band gaps which are sometimes called stop bands. The propagation of waves with definite frequency in these bands is not allowed. The width of this band depends on the incidence angle of light, the indices of refraction of the layers and their thicknesses. If all these parameters are maintained constant, then this one-dimensional photonic crystal structure will possess fixed predetermined forbidden bands of definite frequencies. Electromagnetic waves of frequency lying in these bands will be reflected by the structure.

Photonic crystals can be grouped into categories depending on the number of media in one cell. They can be sorted into binary, ternary, quaternary or so on. This classification is mainly dependent on the number of media in one period. The binary photonic crystal comprises two media in the period, the ternary has three media in a period, the quaternary has four media in a period and so on. These periods are repeated many times to form photonic band gap structures.

Ternary photonic crystal structures have received much more interest because of their excellent performance over the binary photonic crystal structures as sensors for refractometric applications [5], omni-directional reflectors [6–8] and optical filters [9]. The ternary photonic structures may be constructed by the repetition of three materials. The one-dimensional binary photonic crystal can be created by the repetition of two media. The one-dimensional ternary photonic band gap structures exhibit also superior temperature sensing elements [10].

Materials are described by their permittivity ε and permeability μ. Some materials have simultaneous negative electric permittivity and magnetic permeability μ. Those materials have received high interest due to the possibility of being used in many fields. They were referred to as left-handed materials (LHMs) because the electric field **E,** magnetic field **H** and wave vector **K** constitute a left-handed set. LHMs were first studied theoretically by the Russian physicist Veselago in 1968 [11]. They have many properties which are different from normal materials with positive ε and μ. Pendry *et al.* was the first to show the possibility of LHM design by some novel man-made materials [12,13]. Shelby *et al.* fabricated the first LHM in 2001 [14]. Since his wonderful discovery, slab waveguide structures having left-handed material layer have become an interesting topic [15].

Using a slab waveguide as optical sensors is the most recent application of waveguides. They have been utilized as efficient sensors for environmental monitoring, pharmaceutical industry, and food technology. They have many advantages over other sensors such as immunity to light interference [16], fast response, reduced weight and size, resistance to aggressive environments, and easy to interface with optical data communication systems.

One of the advantages of slab waveguides and optical fibers is their immunity to electromagnetic interference. It is well known that all metal cable network media have a common problem since they are at risk of electromagnetic interference. Slab waveguides

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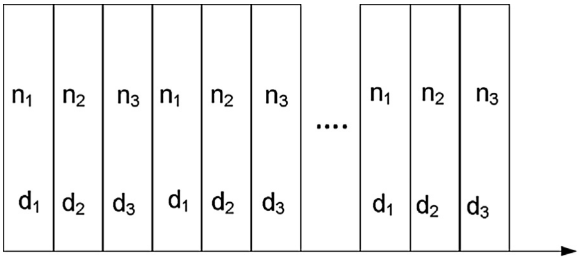

Fig. 1. Ternary photonic crystal with three layers having the indices n_1, n_2 and n_3.

and optical fibers do not suffer from that problem because they are immune to crosstalk since they do not conduct electricity and guide light signals in a dielectric medium rather than electrical signals along a metallic conductor, to transmit data. So they cannot produce a magnetic field result in immune to electromagnetic interference.

In 1989, Tiefenthaler *et al.* studied a slab waveguide for humidity sensing by measuring the change in the effective index of refraction due to any variation in the material contained in the cladding [17]. Since then, considerable research has been conducted to miniature the system and enhance the sensitivity of slab waveguide sensors. Moreover, optical fibers have been used in biosensing in the past few years [18,19]. Surface plasmon resonance (SPR) has been proposed in the field of refractometry [20–22]. Metal-clad waveguide (MCWG) design is one of the most commonly used structures as an optical sensor [23–25]. MCWG structure is the same as a waveguide structure of three layers with an extra metal layer. The metal layer is usually sandwiched between the semi-infinite substrate and the core film. Waveguide sensors are recognized as evanescent field sensors because the evanescent wave is responsible for the sensing operation [26,27]. Any change in the analyte index causes a change in the modal index of the guided mode.

In this work, we report the use of one-dimensional ternary photonic crystal in optical sensing applications. The layers of the ternary photonic crystal are dielectric / LHM / air repeated N times in a periodic structure. The transmission spectrum from the proposed structure is investigated and the wavelength shift due to the change in the refractive index of air is calculated with different parameters of the structure. To the best of my knowledge, this is the first time an LHM is used as a layer in a ternary photonic crystal for optical sensing applications and this is the novel scientific contribution of the current work.

2. Theory

Figure 1 shows a schematic diagram of the proposed photonic crystal. It consists of three layers having the indices n_1, n_2 and n_3 with thicknesses d_1, d_2 and d_3, respectively. The period of the lattice is given by $d = d_1 + d_2 + d_3$. Layer 2 is considered LHM of negative parameters (ε_2, μ_2). The index of refraction n_2 can be calculated as $n_2 = -\sqrt{\varepsilon_2 \mu_2}$.

The characteristic matrix of a cell consisting of three layers is given by

$$\mathrm{M}[\mathrm{d}] = \prod_{i=1}^{l} \begin{bmatrix} \cos\beta_i & \dfrac{-i\sin\beta_i}{p_i} \\ -ip_i\sin\beta_i & \cos\beta_i \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{1}$$

Where $l = 3$, $\beta 1 = \frac{2\pi n_1 d_1 \cos\theta_1}{\lambda_0}$, $\beta_2 = \frac{2\pi n_2 d_2 \cos\theta_2}{\lambda_0}$, $\beta_3 = \frac{2\pi (n_3) d_3 \cos\theta_3}{\lambda_0}$, $p_1 = \sqrt{\frac{\varepsilon_1}{\mu_1}} cos\theta_1$, $p_2 = \sqrt{\frac{\varepsilon_2}{\mu_2}} cos\theta_2$ and $p_3 = \sqrt{\frac{\varepsilon_3}{\mu_3}} cos\theta_3$. and λ_o is the free space wavelength.

θ_1, θ_2 and θ_3 are the angles which the ray makes with the normal to the interfaces in layers 1, 2 and 3, respectively. They are connected to the incidence angle θ_0 through

$$\cos\theta_1 = \left[1 - \frac{n_0^2 sin^2\theta_0}{n_1^2}\right]^{\frac{1}{2}}, \ \cos\theta_2 = \left[1 - \frac{n_0^2 sin^2\theta_0}{n_2^2}\right]^{\frac{1}{2}}$$

$$\text{and } \cos\theta_3 = \left[1 - \frac{n_0^2 sin^2\theta_0}{(n_3)^2}\right]^{\frac{1}{2}}.$$

The derivation of matrix M is provided in Appendix A.

Inspection of the determinant of the matrix M[d] in Eq. (1), we find that $\left|M[d]\right| = 1$.

If the system under consideration has N periods, the characteristic matrix is written as

$$[M(d)]^N = \begin{bmatrix} M_{11}T_{N-1}(a) - T_{N-2}(a) & M_{12}T_{N-1}(a) \\ M_{21}T_{N-1}(a) & M_{22}T_{N-1}(a) - T_{N-2}(a) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{2}$$

where

$$M_{11} = \left(\cos\beta_1\cos\beta_2\cos\beta_3 - \frac{p_2\sin\beta_1\sin\beta_2\cos\beta_3}{p_1} - \frac{p_3\cos\beta_1\sin\beta_2\sin\beta_3}{p_2} - \frac{p_3\sin\beta_1\cos\beta_2\sin\beta_3}{p_1}\right) \tag{3}$$

$$M_{12} = -i\left(\frac{\sin\beta_1\cos\beta_2\cos\beta_3}{p_1} + \frac{\cos\beta_1\sin\beta_2\cos\beta_3}{p_2} + \frac{\cos\beta_1\cos\beta_2\sin\beta_3}{p_3} - \frac{p_2\sin\beta_1\sin\beta_2\sin\beta_3}{p_1 p_2}\right) \tag{4}$$

$$M_{21} = -i\left(p_1\sin\beta_1\cos\beta_2\cos\beta_3 + p_2\cos\beta_1\sin\beta_2\cos\beta_3 + p_3\cos\beta_1\cos\beta_2\sin\beta_3 - \frac{p_1 p_3\sin\beta_1\sin\beta_2\sin\beta_3}{p_2}\right), \tag{5}$$

$$M_{22} = \left(\cos\beta_1\cos\beta_2\cos\beta_3 - \frac{p_1\sin\beta_1\sin\beta_2\cos\beta_3}{p_2} - \frac{p_2\cos\beta_1\sin\beta_2\sin\beta_3}{p_3} - \frac{p_1\sin\beta_1\cos\beta_2\sin\beta_3}{p_3}\right) \tag{6}$$

T_N are well-known functions called Chebyshev polynomials of the second kind

$$T_N(a) = \frac{\sin\left[(N+1)cos^{-1}a\right]}{\left[1 - a^2\right]^{\frac{1}{2}}}, \tag{7}$$

where

$$a = \frac{1}{2}[M_{11} + M_{22}]. \tag{8}$$

Using the matrix elements m_{ij}, the transmission coefficient of the stack of layers can be written as

$$t = \frac{2p_0}{(m_{11} + m_{12}p_0)p_0 + (m_{21} + m_{22}p_0)}. \tag{9}$$

where

$$p_o = n_0\cos\theta_0 = \cos\theta_0. \tag{10}$$

with $n_0 = 1$.

The transmittance is then given by

$$T = |t|^2. \tag{11}$$