Econometrics and Statistics 1 (2017) 2–18

Contents lists available at ScienceDirect

Econometrics and Statistics

journal homepage: www.elsevier.com/locate/ecosta

Structural vector autoregressions with heteroskedasticity: A review of different volatility models

Helmut Lütkepohl[a,*], Aleksei Netšunajev[b]

[a] DIW Berlin and Freie Universität Berlin, Mohrenstr. 58, 10117 Berlin, Germany
[b] Bank of Estonia, Estonia pst. 13, Tallinn 15095, Estonia

ARTICLE INFO

Article history:
Received 20 December 2015
Revised 9 May 2016
Accepted 9 May 2016
Available online 5 November 2016

Keywords:
Structural vector autoregression
Identification via heteroskedasticity
Conditional heteroskedasticity
Smooth transition
Markov switching
GARCH

ABSTRACT

Changes in residual volatility are often used for identifying structural shocks in vector autoregressive (VAR) analysis. A number of different models for heteroskedasticity or conditional heteroskedasticity are proposed and used in applications in this context. The different volatility models are reviewed and their advantages and drawbacks are indicated. An application investigating the interaction between U.S. monetary policy and the stock market illustrates the related issues.

1. Introduction

Structural vector autoregressive (SVAR) models are typically identified by exclusion restrictions on the impact effects of the structural shocks (e.g., Sims (1980), Bernanke and Mihov (1998), Kilian (2009)), by restrictions on the long-run effects of the shocks (e.g., Blanchard and Quah (1989), King et al. (1991)) or by restrictions on the signs of the responses of specific variables to a shock (e.g., Faust (1998), Canova and De Nicoló (2002), Uhlig (2005)). Recently, a number of articles uses changes in the volatility of the variables for identifying SVAR models (e.g., Rigobon (2003)). A number of alternative approaches have been proposed for modeling the changes in volatility. For example, exogenous changes in the variances of the residuals are considered by Rigobon (2003), Rigobon and Sack (2003) and Lanne and Lütkepohl (2008). In contrast, Lanne et al. (2010) and Herwartz and Lütkepohl (2014), for example, model the changes in volatility by a Markov regime switching (MS) mechanism whereas Lütkepohl and Netšunajev (2014b) consider a smooth transition of the residual volatility from one regime to another. Yet another approach is based on a generalized autoregressive conditional heteroskedastic (GARCH) error structure (e.g., Normandin and Phaneuf (2004), Bouakez and Normandin (2010), Weber (2010), Strohsal and Weber (2015)). Typically in this literature there is not much discussion why a particular volatility model is used.

In this study we compare the alternative approaches and discuss their advantages and drawbacks. Specifically we discuss the model setup, identification conditions for the shocks, estimation and inference methods related to the alternative models. As far as estimation and inference is concerned, we focus on frequentist procedures because Bayesian procedures have not been applied much so far in this context (see, however, Kulikov and Netšunajev (2013) and

* Corresponding author.
E-mail addresses: hluetkepohl@diw.de (H. Lütkepohl), Aleksei.Netsunajev@eestipank.ee (A. Netšunajev).

http://dx.doi.org/10.1016/j.ecosta.2016.05.001
2452-3062/

Wozniak and Droumaguet (2015)). Hence, there is not much experience with Bayesian methods. For some of the models they are not yet fully developed.

We illustrate the different approaches by reconsidering a SVAR study by Bjørnland and Leitemo (2009). These authors use conventional exclusion restrictions on the impact and long-run effects of the shocks in studying the interaction between monetary policy and the stock market in the U.S. We demonstrate how the different volatility models can be used for generating additional identifying information and we discuss the advantages and drawbacks of the alternative models for the example system.

Related earlier reviews of some of the models discussed in the following are given by Lütkepohl (2013) and Lütkepohl and Velinov (2016). They discuss a more limited set of volatility models and different, more limited examples. In particular, they do not compare the full range of volatility models in the context of a specific example. The latter of the two articles focuses specifically on combining restrictions on the long-run effects of the shocks with identifying information from changes in volatility. That topic is part of our general setup and, hence, it is included in the present study as a special case.

The remainder of this study is organized as follows. In the next section the basic SVAR model is introduced and the different volatility models are discussed in Section 3. The illustrative example is considered in Section 4 and concluding remarks are given in the final section.

2. The baseline model

The vector of time series variables of interest is K-dimensional and is denoted by $y_t = (y_{1t}, \dots, y_{Kt})'$. The reduced-form of the data generation process (DGP) is a VAR model of order p (VAR(p)),

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \tag{1}$$

where ν is a $(K \times 1)$ constant intercept term, the A_i are $(K \times K)$ coefficient matrices and the error term u_t is a K-dimensional white noise process with mean zero and nonsingular covariance matrix Σ_u.

The structural errors, denoted by ε_t, are obtained from the reduced-form residuals by a linear transformation:

$$\varepsilon_t = B^{-1} u_t. \tag{2}$$

Writing the transformation in this way indicates that it is nonsingular and, using the relation $u_t = B\varepsilon_t$ the matrix B is easily seen to contain the instantaneous effects of the structural shocks on the observed variables. The process ε_t is also a serially uncorrelated white noise process with mean zero. Typically the components are also assumed to be instantaneously uncorrelated, that is, $\varepsilon_t \sim (0, \Sigma_\varepsilon)$, where Σ_ε is a diagonal matrix. Often it is convenient to choose B such that the variances of the structural shocks are normalized to one. In other words, Σ_ε is an identity matrix.

Given the relation between the reduced form residuals and the structural shocks, the matrix B has to satisfy $\Sigma_u = B\Sigma_\varepsilon B'$. Put differently, in order to qualify as transformation for getting the structural errors from the reduced-form errors in (2), the matrix B has to be such that $\Sigma_u = B\Sigma_\varepsilon B'$. Without further restrictions the matrix B and, hence, the structural innovations are not uniquely determined.

Identifying (uniqueness) restrictions are traditionally imposed on B or its inverse directly. Typical restrictions are zero restrictions on the impact effects of shocks on some of the variables (Sims, 1980) or restrictions on the long-run effects of structural shocks as in Blanchard and Quah (1989). For stable, stationary processes the matrix of accumulated long-run effects of structural shocks given by

$$\Xi_\infty = (I_K - A_1 - \cdots - A_p)^{-1} B$$

is considered for this purpose, whereas for integrated and cointegrated processes the long-run effects matrix is related to the cointegration structure of the model (see, e.g., Lütkepohl (2005) or Lütkepohl and Velinov (2016)). In any case, it is important to observe that the matrix of long-run effects can be computed from the reduced-form and structural parameters. Given the DGP and, hence, the reduced-form, imposing restrictions on the matrix of long-run effects implies restrictions on B.

In the SVAR literature it is not uncommon to impose as few restrictions as possible to achieve identification of the structural parameters B. In other words, the structural model and the structural shocks are just-identified. Competing sets of just-identifying restrictions correspond to identical reduced forms and cannot be tested against the data. Thus, in the conventional setup it is often not possible to let the data discriminate between competing economic models or theories. In the next section it is discussed how heteroskedasticity can be used to let the data speak on competing models that are just-identified in a conventional framework.

3. SVAR models with time-varying volatility

3.1. Identifying structural shocks

Suppose now that u_t is a heteroskedastic or conditionally heteroskedastic error term. This means that the variances or conditional variances of the reduced form and, hence, also the structural innovations change over time. In the macroeconomic literature such behaviour is well documented. Allowing for this feature means that there is more than one volatility