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Material homogenization technique for composites: A meshless formulation

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Abstract

The analysis of the structural behaviour of heterogeneous materials is a topic of research in the engineering field. Some heterogeneous materials have a macro-scale behaviour that cannot be predicted without considering the complex processes that occur in lower dimensional scales. Therefore, multi-scale approaches are often proposed in the literature to better predict the homogeneous mechanical properties of these materials. This work uses a multi-scale numerical transition technique, suitable for simulating heterogeneous materials, and combines it with a meshless method – the Radial Point Interpolation Method (RPIM) [1]. Meshless methods only require an unstructured nodal distribution to discretize the problem domain. In the case of the RPIM, the numerical integration of the integro-differential equation from the Galerkin weak form is performed using a background integration mesh. The nodal connectivity is enforced by the overlap of influence-domains defined in each integration point. In this work, using a plane-strain formulation, representative volume elements (RVE) are modelled and periodic boundary conditions are imposed on them. A computational homogenization is implemented and effective elastic properties of a composite material are determined. In the end, the solutions obtained using the RPIM and also a lower-order Finite Element Method are compared with the ones provided in literature.

Keywords: Micromechanics; Composite materials; Homogenized elastic properties; Meshless methods; Radial Point Interpolation Method (RPIM)

1. Introduction

Composite structures have been used in the engineering field in the past few decades and settled as materials for primary structural components in industries such as aircraft, aerospace or automotive. Their high specific mechanical properties and low weight make them ideal solutions for these applications. Thus, their structural analysis needs to be accurate and efficient.

The analysis of a composite material can be performed according to two different approaches that are often coupled in numerical frameworks: the macromechanical analysis and the micromechanical analysis. In macromechanics, the composite structure is considered as an homogeneous orthotropic continuum [2]. In this case, it is not considered the micro-heterogeneity of the composite material, which is a sufficient consideration for the majority of the engineering applications. Nevertheless, for more complex situations (such as micro-cracking or buckling of fibres), where the microscopic phenomena considerably influences the behaviour of the material at the macroscale, the micromechanical analysis is a more appropriate approach [3]. Thus, distinct multi-scale approaches are often found in the literature considering coupled analysis at different scales.

This work proposes a new micromechanical computational tool capable of being applied to a wide range of heterogeneous materials. This research is based on existing multi-scale numerical transition techniques [3–9] suitable for simulating heterogeneous materials assuming different arrangements of fibres in a fibre composite material and Representative Volume Elements (RVE) statistically representing the microstructure of the material and containing the information of the elastic constants and fibre volume fraction of the composite material. Using the scale transition theory, a homogenization procedure can be implemented at each infinitesimal point of the macro-scale (represented by the RVE), and homogenized elastic properties can be determined. These micromechanical principals are implemented within the formulation of an advanced dis-

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cretization technique – the Radial Point Interpolation Method – and also a lower order Finite Element Method (FEM).

2. Meshless methods

In meshless methods, the problem domain is discretized in a set of nodes which are intrinsically independent from each other. Unlike the FEM, in meshless methods the field variables are approximated within an 'influence-domain' rather than an element [1]. It is also the overlap of influence-domains that ensures the nodal connectivity in these methods. 'Influence-domains' are areas or volumes (if the considered problem is bidimensional or tridimensional, respectively) concentric with an interest point [1], containing a certain number of nodes. The high nodal connectivity, and the fact that they are not mesh reliant, makes meshless methods advanced discretization techniques that are solid alternatives to the FEM.

As regards the integration mesh, it be can nodal dependent or independent, being the last case called a 'not-truly' meshless method since the mesh-free characteristic of these methods is not verified in that case (the method requires a background integration mesh). In comparison to the FEM, in meshless methods the shape functions have virtually a higher order, allowing a higher continuity and reproducibility [1]. Additionally, meshless methods can easily handle situations where the geometry is transitory such as problems involving large deformations or fracture mechanics – which are frequently associated, in the FEM, with re-meshing procedures showing high computational costs.

Another advantage of meshless methods concerns the simplified refinement procedure of the nodal mesh, which can be easily changed (by adding or removing nodes) [10]. Also, the solutions obtained from the meshless methods can be more accurate when compared with a lower order FEM [10], as will be shown in this study.

The first proposed meshless method was the Smooth Particle Hydrodynamics Method (SPH) [11] in 1977, but the first global weak form based meshless method was only presented in 1994 with the Element Free Galerkin Method (EFGM) [12]. From that date, other meshless methods were presented in literature such as the Meshless Local Petrov–Galerkin Method (MLPG) [13], the Reproducing Kernel Particle Method (RKPM) [14], Point Interpolation Method (PIM) [15], the Point Assembly Method [16], the Radial Point Interpolation Method (RPIM) [17] or the Natural Neighbour Radial Point Interpolation Method (NNRPIM) [1,18]. These meshless methods have been used in the past decades in several engineering applications and prove to be reliable and accurate numerical tools.

In the past years, some of the mentioned meshless methods were used in the field of computational homogenization and multi-scale approaches. For instance, the RPIM was used to impose periodic boundary conditions for the representative volume element (RVE) model of a 3D braided composite by Wang et al. [19]. The determination of homogenized elastic properties was a topic of interest of Falzon and Muthu [20] when using the EFGM for constructing an RVE. Wang and Liew [21] used a meshless method based on the moving least-squares (MLS) approximation and the EFGM for a micromechanical study of carbon nanotubes; Ahmadi et al. [22] used a truly meshless method based on the integral form of energy equation to study the steady-state heat conduction in an RVE of anisotropic and heterogeneous materials. Ahmadi and Aghdam [22], using the same meshless method previously mentioned, studied microstresses in a unidirectional fibre reinforced composite material which is subjected to a normal and a shear load; Rastkar et al. used an asymptotic homogenization and a meshfree Solution Structure Method (SSM) to obtain homogenized material properties; Yang et al. [23] presented a multiscale method for crack propagation using a Meshfree Adaptive Multiscale Method for Fracture (MAMMF).

In this work, the RPIM formulation is used for the micromechanical analysis of composite materials using RVEs defined in a plane-strain state.

2.1. The Radial Point Interpolation Method

The RPIM was created from the Point Interpolation Method (PIM) [15]. Using the polynomial basis function (as the PIM) and also introducing an additional radial basis function (RBF), the interpolation functions of the RPIM become more stable and robust than the ones used in the PIM.

This meshless method has been used in inelastic analysis of 2D solids [24], 3D contact problems [25], crack growth modelling in elastic solids [26], non-local constitutive damage models or even the bending [10,27] and dynamic [28–30] analysis of composite plates and shells. In this study, the RPIM will be used for micromechanical analyses of composite materials.

In the following subsections, the formulation of the RPIM is presented.

2.1.1. Influence-domain concept. Nodal connectivity

As it was stated before, the nodal connectivity in the RPIM is imposed by an overlap rule between 'influence-domains', created following the nodal discretization (which can be regular or irregular, with the last case having, in general, lower accuracy) and the position of the integration point from the integration mesh. The influence-domain can have a fixed size (containing a well-defined number of nodes closer to a certain interest point) or a variable size, which consists in creating circles or rectangles concentric with an interest point and the nodes placed within those areas belong to the influence-domain of that interest point – Fig. 1. The literature recommends [1] the use

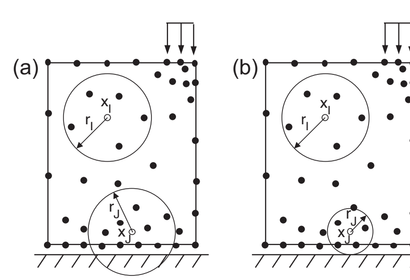

Fig. 1. (a) Variable size influence-domain ($r_I = r_J$ but the number of nodes within each influence-domain is different); (b) Fixed size influence-domain ($r_I \neq r_J$ but the number of nodes within each influence-domain is the same).