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Non-probabilistic interval process analysis of time-varying uncertain structures

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A B S T R A C T

In the engineering practices, the information or the sample data to construct the precise probabilistic characteristics are usually insufficient. Aim at this issue, a non-probabilistic interval process model, whose upper and lower bounds can be determined on the basis of limited data, is introduced to describe the time-varying uncertainties. To obtain the dynamic response bounds of the structure under the non-probabilistic interval process model, a numerical method is proposed, namely the interval Chebyshev surrogate model based on the Karhunen-Loève expansion (ICSM-KLE), are proposed. In this method, the time-dependency between the adjacent values of an interval process is quantified by the Karhunen-Loève expansion. And then, the structural dynamic response bounds of the time-varying uncertain structure is respectively approximated by the Chebyshev surrogate model. Two numerical examples, including a multi-degree-of-freedom linear vibration system and a continuum shell structure, verify that the accuracy of the ICSM-KLE is very high, when compared with the referenced results provided by the direct Monte Carlo method. Thus, the ICSM-KLE provides a good platform for the non-probabilistic interval process analysis of the time-varying uncertain structures with limited information.

1. Introduction

Due to the aggressive environmental factors, the manufacturing and assembling errors, the material degenerations and losses, the unpredictable exterior excitations and so on, uncertainties unavoidably exist in various engineering structures. Without considering these ubiquitous uncertainties, the response prediction and the optimal design of a structure may fail. Up to now, the main nondeterministic models to deal with the structural uncertainties are the stochastic variable model and the stochastic process model [1–5]. The first one is developed for the time-independent uncertain parameters. The second one is proposed for the time-varying uncertain parameters. Based on these nondeterministic models, various of numerical methods such as the Monte Carlo method, the stochastic perturbation method and the spectral stochastic method have been proposed.

The direct Monte Carlo method is the most robust method for the response analysis of stochastic structures [6–9]. Because of its probability convergent characteristics, the Monte Carlo method with a large number of samples can produce an excellently precise solution for the stochastic problem. The critical defect of the Monte Carlo method is its fatally computational burden, especially for large-scale engineering structures. To greatly improve the computational efficiency of the Monte Carlo method without deteriorating its accuracy, several variants including an important sampling Monte Carlo method [10], a subset simulation Monte Carlo method [11–13] and a line sampling Monte Carlo method [14,15] have been developed. The accuracies of these variants significantly depend on their sampling rules. The stochastic perturbation method is another effective numerical method for the response analysis of the stochastic structure. Culla and Carcaterra [16] proposed a conventional perturbation-statistical perturbation method (CPSPM) and a statistical linearization-statistical perturbation method (SLSPM) for the investigation of the statistical moment of a floating body excited by random waves. Kamiński [17,18] developed a stochastic perturbation method to predict the probabilistic moments of the stochastic structure with the Gaussian elastic modulus. Muscolino et al. [19–21] have systematically investigated the dynamic response of the deterministic and nondeterministic structures under the Gaussian stochastic excitations. Xia et al. [22] proposed a transformed perturbation stochastic method based on a change of variable technique for the response analysis of the stochastic systems. Do et al. [23,24] creatively constructed a multiple random field model and provided a stochastic Galerkin scaled finite element method for the response analysis of the stochastic structures. Wu and Zhong [25] have predicted the dynamic response of the stochastic structure with uncorrelated or correlated

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random variables. Zhang et al. [26,27] developed a stochastic level set perturbation for the robust topology optimization of the stochastic structure with the uncertain geometric parameters and the uncertain harmonic excitations.

On the whole, the stochastic numerical method is a very mature method for the uncertain problems with enough information. However, in many engineering practices, it is extremely cost and even impossible to obtain sufficient sample data to accurately evaluate the statistical characteristics of uncertain parameters. When the estimated probabilistic parameters deviate from their real values, the responses of uncertain structures may become unreliable, which will provide a disastrous guide in the design of engineering systems. Under this circumstance, the interval model is an important alternative because of its excellent ability to deal with uncertainties with limited sample data [28]. In the interval model, the uncertain parameters are treated as the interval variables or the interval fields whose variational ranges are well defined. Based on the interval model, several numerical methods have been developed. Qiu et al. [29,30] proposed an interval perturbation method for the response analysis of the structures and the nonlinear systems with interval uncertainties. Xia et al. [31,32], by introducing a modified Neumann expansion with the higher order terms, improved the accuracy of the interval perturbation method. Wang et al. [33–36] extended the modified interval perturbation method for the investigation of the exterior acoustic field, the steady-state temperature field, the steady-state heat convection-diffusion problem and the eigenvalues of structure. Luo et al. [37–39] proposed a non-probabilistic reliability-based topology optimization for the geometrically nonlinear structures and the two-material structures. Wu et al. [40–42] proposed a Chebyshev surrogate model for the nonlinear dynamic systems, the multibody mechanical systems and the structural topology optimization with interval uncertainties. Sofi et al. [43–47] introduced the interval field model for the description of spatially dependent uncertainties. Gao et al. [48–54] have systematically investigate the linear and nonlinear structures under the interval field model and proposed hybrid uncertain analysis method under hybrid random and interval field model. Xu et al. [55–57] proposed an orthogonal polynomial chaos expansion for the uncertain propagation of the structural-acoustic systems or the structural vibration systems with interval parameters. In addition to the above method, Wang et al. [58–61] developed a Bayesian approach for characterizing the site-specific joint probability distribution of important soil parameters in evaluating stability and deformation of geotechnical structures.

The above interesting interval numerical methods are based on the time-invariant interval models in which the variational ranges of uncertain parameters are constant in the considered time periods. Unfortunately, for the most practical problems, parameters related with the degrading material properties and the dynamic loads are always time-varying. These time-varying uncertainties play significant roles on the whole-life safety assessment of the engineering system. Without considering these time-varying uncertainties, a disastrous consequence may be yielded. To describe the time-varying uncertainties with the limited information, Jiang et al. [9,62,63] proposed a non-probabilistic interval process model and introduced a Monte Carle method to predict the dynamic response of the structure. The dynamic responses of the time-varying uncertain structures are derived as the non-probabilistic interval processes with the maximal and minimal response bounds, which can then provide a significant information for the reliability-based design of the dynamic engineering structures [64–67].

In this paper, we will investigate the dynamic response of the time-varying uncertain structures. First, the non-probabilistic interval process model is introduced to describe the time-varying uncertainties with limited information. And then, the Karhunen-Loève (KL) expansion is employed to quantify the time-dependency of the non-probabilistic interval process. Furthermore, a numerical method, named as the interval Chebyshev surrogate model based on the KL expansion (ICSM-KLE), is proposed. In the ICSM-KLE, the dynamic response of the structure is approximated by the Chebyshev surrogate model with a high computational efficiency.

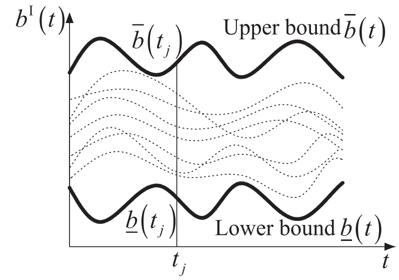

Fig. 1. A non-probabilistic interval process.

2. Non-probabilistic interval process model based on KL expansion

In many engineering practices, the information to determine the precise probabilistic characteristics of the time-varying uncertainties is usually insufficient. In this case, a non-probabilistic interval process model (shown in Fig. 1) will be introduced to describe the uncertainty of a time-varying parameter.

As is shown in Fig. 1, a non-probabilistic interval process can be expressed as $b^{\mathrm{I}}(t) = [\underline{b}(t), \bar{b}(t)]$, in which $\underline{b}(t)$ and $\bar{b}(t)$ represent the lower and upper bounds of the non-probabilistic interval process $b^{\mathrm{I}}(t)$. It should be noted that $\underline{b}(t)$ and $\bar{b}(t)$ are functions of the time t. At any time t_j, the uncertainty of the non-probabilistic interval process $b^{\mathrm{I}}(t)$ can be expressed as an interval variable $b^{\mathrm{I}}(t_j)$ with a lower bound $\underline{b}(t_j)$ and an upper bound $\bar{b}(t_j)$. Generally, the interval process is treated as a series of interval variables and the time dependency between adjacent interval variables are neglected. The KL expansion has been successfully used to track the time dependency between adjacent values of a stochastic process. In this section, the KL expansion is further developed to quantify the time dependency of a non-probabilistic interval process.

A dimensionless interval process function $d^{\mathrm{I}}(t) = [\underline{d}(t), \bar{d}(t)]$ is defined. The midpoint value is zero and the uncertainty is $\Delta d(t) \leqslant 1$. Based on this dimensionless interval function, the non-probabilistic interval process $b^{\mathrm{I}}(t)$ can be defined as follow:

$$b^{\mathrm{I}}(t) = b^{\mathrm{m}}(t)[1 + d^{\mathrm{I}}(t)] \tag{1}$$

where the midpoint value $b^{\mathrm{m}}(t)$, the variational range $\Delta b(t)$ of $b^{\mathrm{I}}(t)$ and $\Delta d(t)$ are given as

$$\begin{aligned} \mathrm{mid}\,\{b^{\mathrm{I}}(t)\} &= \frac{\bar{b}(t) + \underline{b}(t)}{2} \equiv b^{\mathrm{m}}(t) \\ \Delta b(t) &= \frac{\bar{b}(t) - \underline{b}(t)}{2} \equiv b^{\mathrm{m}}(t)\Delta d(t) \\ \Delta d(t) &= \frac{\bar{d}(t) - \underline{d}(t)}{2} \end{aligned} \tag{2}$$

where mid$\{\cdot\}$ expresses the midpoint of the non-probabilistic interval process function.

The time-dependency of a non-probabilistic interval process should be governed by a real, symmetric, non-negative, deterministic and bounded $f(t, t')$ function, defined as

$$f(t, t') = \mathrm{mid}\{d^{\mathrm{I}}(t)d^{\mathrm{I}}(t')\} \equiv \frac{\mathrm{mid}\{b^{\mathrm{I}}(t)b^{\mathrm{I}}(t')\}}{(b^{\mathrm{m}}(t))^2} - 1 \tag{3}$$

The time-dependency function $f(t, t')$ can be considered as a non-probabilistic counterpart of the auto-correlation function in the stochastic process. By analogy with the classic stochastic process model, we can construct an interval process model. To gain a further insight into this concept, we firstly review a homogeneous Gaussian stochastic process, defined as

$$\tilde{b}(t) = b^{\mathrm{m}}(t)[1 + \tilde{d}(t)] \tag{4}$$