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Two-parameter kinematic theory for punching shear in reinforced concrete slabs without shear reinforcement

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ABSTRACT

Measurements taken from recent test series with varying slenderness reveal strong differences between fracture kinematics of slender and compact slabs failing in punching. In accordance with the assumptions of the existing kinematic punching shear resistance models, the deformation behavior of slender slabs is governed by flexural deformations. However, in very compact slabs only small flexural deformations occur and the deformation behavior is dominated by translational deformations. In the transition region between slender and very compact slabs, the deformation behavior is influenced by both deformation components. As a consequence, the general application of the existing kinematic models to both slender and compact slabs might yield unexpected results.

In this paper, a two-parameter kinematic theory for punching shear in reinforced concrete slabs without shear reinforcement is developed taking into account the aforementioned observations. In the theory, it is assumed that shear forces are transmitted along the failure crack by four shear contributions, namely the contributions of compression ring, aggregate interlock, residual tensile stresses, and dowel action. The magnitude of shear contributions is estimated based on the deformed slab accounting for two degrees of freedom (DOFs). While the first DOF accounts for flexural deformations, the second DOF considers translational deformations. Subsequently, the punching strength is calculated by summation of the contributions. The evaluation of the proposed theory by means of systematic test series and databanks yields good agreement between predictions and experimental results. Especially, the differences between flat slabs and column bases can be explained in a consistent manner by the theory.

1. Introduction

The problem of punching of reinforced concrete slabs has been dealt with extensively in literature (e.g. [1–5]). While most of the existing punching tests have been performed on flat slab specimens, fewer test series have been conducted on column bases (e.g. footings [6–15]). As a consequence, the majority of the existing punching shear resistance models were derived considering mainly the test results from flat slab specimens. Nevertheless, since the punching shear behavior of flat slabs differs significantly from column bases [10,11,16], a general application of the existing models to both flat slabs and column bases might lead to unexpected results.

To allow for a more general description of the punching shear behavior of reinforced concrete slabs, the knowledge of slab deformations at punching failure is necessary. Measurements taken from systematic test series with varying slenderness reveal strong differences between the fracture kinematics of slender and compact reinforced concrete slabs failing in punching [17]. While the deformation behavior of slender slabs is governed by flexural deformations, the deformation behavior of very compact slabs is dominated by translational deformations. In the transition region between slender and very compact slabs, the deformation behavior can be described by both deformation components.

Evaluations of punching tests with varying slenderness indicate that a general description of the fracture kinematics of reinforced concrete slabs is possible by introducing two degrees of freedom (DOF) [17]. The first DOF is the average strain in the flexural reinforcement $\varepsilon_{t,avg}$, which is proportional to the slab rotation ψ and, thus, to the flexural deformation δ_f. The second DOF is the translational deformation δ_t caused by shear deformation and column penetration. The total deformed shape of reinforced concrete slabs failing in punching can be obtained by superimposing the two DOFs $\varepsilon_{t,avg}$ and δ_t as shown in Fig. 1(a). The kinematic principles presented in [17] were derived based on the work by Mihaylov et al. for deep beams [18,19] and shear walls [20].

Based on the kinematic considerations presented in [17], a two-parameter kinematic theory for punching shear in reinforced concrete

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Notation

Roman lower case letters

a_s	amount of flexural reinforcement per unit length
a_λ	shear span
c	column dimension
d	effective depth
d_g	maximum aggregate size
f_c	concrete compressive strength
f_{ct}	concrete tensile strength
f_y	yield strength of flexural reinforcement
h	depth
$h_{c,ef}$	effective concrete depth
l	length
l_k	length over which δ_t develops
l_t	length over which $\varepsilon_{t,avg}$ is averaged
r	radius
r_c	column radius
r_{cr}	horizontal length of compression ring
r_{pc}	radius of punching cone
r_q	radius of moment contraflexure
r_s	slab radius
s	tangential crack opening
s_1	crack slip associated with translational DOF δ_t
u	perimeter
u_0	perimeter of loaded area
u_n	net perimeter on the level of the flexural reinforcement
w	normal crack opening
w_c	critical crack opening at which tensile stresses no longer can be transmitted
w_0	crack width associated with flexural DOF $\varepsilon_{t,avg}$
w_1	crack width associated with translational DOF δ_t
x	depth of compression zone
x_m	sample mean
z	inner lever arm

Roman upper case letters

A_x	projection of area of contact in x-direction
A_y	projection of area of contact in y-direction
E_c	Young's modulus of concrete
E_s	Young's modulus of flexural reinforcement
F_c	force in compression zone
F_s	force in flexural reinforcement
$F_{\sigma,ai}$	resulting normal force from aggregate interlock
$F_{\sigma,rt}$	resulting normal force from residual tensile stresses
$F_{\tau,ai}$	resulting tangential force from aggregate interlock
G_f	fracture energy
V	shear force
V_{ai}	contribution of aggregate interlock
V_{calc}	predicted punching strength
V_{cr}	contribution of compression ring
V_{da}	contribution of dowel action
V_m	shear force derived from moment equilibrium
V_v	shear force derived from vertical equilibrium
V_{rt}	contribution of residual tensile stresses
V_{test}	failure load from punching test
V_x	sample COV

Greek letters

α	inclination of shear crack
δ_f	flexural deformation
δ_t	translational deformation (DOF of kinematic model)
ε	strain
ε_c	strain in concrete
ε_s	strain in flexural reinforcement
ε_t	tangential strain
$\varepsilon_{t,avg}$	average strain in flexural reinforcement (DOF of kinematic model)
ϕ	direction of principle stresses
η_c	factor accounting for brittle behavior of concrete
η_ε	factor accounting for presence of transverse strains in concrete
μ	friction coefficient
ρ_l	flexural reinforcement ratio
σ	normal stress
σ_1	principle stress
σ_2	principle stress
σ_3	principle stress
σ_{ai}	normal stress resulting from aggregate interlock
σ_c	stress in concrete
σ_{pu}	normal stress at projected contact line
σ_{rt}	normal stress resulting from residual tensile stresses
σ_s	stress in flexural reinforcement
σ_x	horizontal stress
σ_z	vertical stress
τ	shear stress
τ_{ai}	shear stress resulting from aggregate interlock
τ_{zx}	shear stress
ψ	slab rotation

Other symbols

$\varnothing$	diameter of flexural reinforcement

slabs without shear reinforcement is developed. The deformed shape of slabs failing in punching is estimated by means of the two DOFs $\varepsilon_{t,avg}$ and δ_t. The magnitude of shear transfer actions occurring along the critical shear crack is calculated based on the deformed shape. The punching shear resistance is calculated by summation of the different shear transfer actions. This paper describes the derivation of the two-parameter kinematic theory as well as its validation by means of systematic punching test series on flat slabs and footings without shear reinforcement. Also, the kinematic theory is evaluated using a databank containing punching tests on flat slabs and footings.

2. Considered shear transfer actions

2.1. General

For the derivation of the two-parameter kinematic theory for punching shear in reinforced concrete slabs without shear reinforcement presented in this paper, the following four shear transfer actions (Fig. 1(b–d)) are considered:

- Contribution of compression zone along the loaded area (compression ring) V_{cr}
- Contribution of aggregate interlock along the critical shear crack V_{ai}
- Contribution of residual tensile stresses along the critical shear crack V_{rt}
- Contribution of dowel action caused by reinforcement bars in