Journal of Sound and Vibration 435 (2018) 104–118

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Fundamental frequencies and buckling in pre-stressed parabolic arches

U. Eroglu [a], G. Ruta [b,*]

[a] Istanbul Technical University, Turkey
[b] University "La Sapienza", Roma, and National Group of Mathematical Physics, Italy

ARTICLE INFO

Article history:
Received 31 January 2018
Revised 20 July 2018
Accepted 24 July 2018
Available online XXX
Handling Editor: Erasmo Carrera

Keywords:
Parabolic arches
Vibration
Buckling
Pre-stress
Galërkin method
Differential quadrature method

ABSTRACT

We operate a perturbation approach on the finite field equations for clamped slender arches with compact symmetric cross-section and parabolic centre curve under a uniform line load parallel to their symmetry axis. We study small vibration superposed on the relevant stress, assumed of membrane nature. We find the fundamental angular frequency in terms of the aspect ratio of the arch and of the pre-load; the possibility of buckling is examined. This is a first step towards monitoring such structures, and evaluating pre-loads and structural integrity by dynamic measurements.

1. Introduction

Arches are fundamental in structures, known since Etruscan, Greek, and Roman architecture, when they were made of trapezoidal chunks of stone suitably arranged; they are widely used also nowadays in many fields of engineering and architecture, employing reinforced concrete, metal, or composites in order to have homogeneous enough, or suitably non-homogenous, structural elements.

One-dimensional models suffice to describe the behaviour of arches when their transverse cross-sections have characteristic dimensions much smaller than the length of the arch span. In addition, the standard hypothesis of transverse cross-sections remaining plane is reasonable, and beam-like theories may satisfactorily be used, only taking care of the initial curved geometry of the axis. In this respect, the first 'rational' theory of curved beams may be dated back to the well-known monographs by Love [1] and Timoshenko [2], while a more recent and thorough presentation of a theory of beams as one-dimensional continua with axis of any shape is by Antman [3,4]. Many contributions were published on the behaviour of arches and curved beams, starting from the search of analytical and/or numerical solutions to linear elastic problems [5–9], then moving forward to investigating the effects of geometric and/or constitutive non linearities [10–18], the stability of solutions [19–21], and the response to local damage [22–26]. Even though we limit to cite some of the main papers in these fields, dedicated readers are referred to the

* Corresponding author.
E-mail addresses: ueroglu@itu.edu.tr (U. Eroglu), giuseppe.ruta@uniroma1.it (G. Ruta).

https://doi.org/10.1016/j.jsv.2018.07.038
0022-460X/

review articles [27,28] and to Karnovsky's monograph [29].

As for other structural elements in all fields of architecture and engineering, dynamic measurements are proposed in the literature for structural monitoring and identification, and for possible damage detection. These techniques are attractive for their non-destructive feature and for their wide range of application. We may refer to the paper by Adams et al. [30] as one of the first to consider the subject, and to the reviews [31,32] for applications in civil and mechanical engineering; instances of analogous investigations in such challenging applications as aerospace engineering may be found, for instance, in the paper [33] and in the recent monographs [34,35].

Despite the fact that arches are frequently used in bridges and arch dams (in architecture and civil engineering) and in many curved elements (in mechanical, industrial, aerospace engineering), most of the studies investigate the free vibration of arches in a so-called natural configuration, i.e., neglecting the effects of possible pre-loads. In the early works on pre-loaded arches [36,37] the interest is in the lowest in-plane and out-of-plane frequencies, and in the critical loads of circular arches with flexible supports. Plaut and Johnson [38] investigate the effects of initial thrust and elastic foundation on the vibration characteristics of pinned-pinned shallow arches with a sinusoidal axis. A comprehensive investigation on the in-plane vibration of pre-loaded circular arches and on the buckling of parabolic arches is in Chidamparam's Ph.D. thesis [39] through Galërkin's approach. Further investigations are performed on the effect of the centre line extensibility on vibration characteristics of circular arches by Chidamparam and Leissa [40], where they conclude that the error due to the non-extensibility constraint on centerline becomes remarkable for shallow arches with a low slenderness ratio. Nieh et al. [41] consider the plane vibration and stability of elliptic arches under a vertical distributed load, derive the differential equations governing the problem by a variational principle accounting for the initial stresses, and admit the solution to be expressed as a power series expansion of the unknown functions. By the same approach, Huang et al. [42] examine a similar problem for shear-deformable circular arches, discussing possible simplifications on the equations and their drawbacks in detail. As for arches of inhomogeneous materials, which gained attention recently, the papers by Kiss and Szeidl [43,44], dealing with circular arches with different boundary conditions subjected to a radial force concentrated at the apex, deserve attention. In addition, the studies accounting for the pre-stresses due to thermal effects in heterogeneous arches [45–47] should be mentioned. Apart from those papers focusing on beam structures with initial curvature, we may quote some investigations on arches obtained starting from large deflected straight beams [48–51], or considering straight beams in a post-buckling regime [51,52].

From this short literature review on the subject, it appears that the great part of the published investigations is on plane circular arches, since their initial uniform curvature brings major simplifications in the field equations of the problem. Due to their importance in many applications, plane parabolic arches received some attention, at least for linear elastic static and dynamic problems.

As far as we know, there is no study on the dynamics of pre-loaded parabolic arches, which is the aim of this contribution. We briefly sketch the finite, non-linear field equations of a parabolic arch, and operate a perturbation approach on them. We show that a state of membrane initial stress is balanced with a 'vertical' load uniformly distributed with respect to the length of the span. Such a state is not the only possible one, since arches are usually statically undetermined - actually, we deal with doubly clamped arches as a benchmark. However, a membrane stress state always exists and is physically reasonable: indeed, it is the unique static solution for arches with supports tangent to their centre curve at their ends, and different external constraints under the same external load will not change the membrane stress state, apart from narrow portions near the ends. A similar behaviour is well known in axisymmetric shells of revolution, where possible bending effects are limited to narrow stripes next to the constrained boundary.

The perturbation approach lets us study small vibration about this pre-stressed configuration, assuming the axis of the arch to be inextensible, and shearing strain between the axis and the cross-sections to vanish. Thus the arch is a purely flexible curved beam, described by the standard Euler-Bernoulli model, which is reasonable for the considered ratios of crown height to span length.

We present the equations governing the problem, suitably turning them into a non-dimensional form to abstract from the actual numerical values of the physical and geometrical quantities involved. The governing equations may be written with respect to either the curvilinear abscissa and the one along the span; in both cases, the problem does not exhibit closed-form solutions, thus, approximated and purely numerical solutions will be searched for. We then present and thoroughly comment the results for the natural angular frequencies for the considered structural element. The condition of vanishing of the fundamental frequencies provides a well-known condition for static bifurcation and will also be discussed.

2. A perturbation approach for pre-stressed parabolic arches

Let a segment of parabola, symmetrical with respect to its vertex, represent the axis of a purely flexible arch, described as a curved beam by the Euler-Bernoulli model. The reference configuration of the body is filled by the copies of a prototype plane region (the transverse cross-section), each attached to the axis parallel to the unit normal at that point, see Fig. 1.

If x, y are Cartesian coordinates in the plane where the axis of the arch lies, with relevant orthogonal basis of unit vectors $\mathbf{e}_1, \mathbf{e}_2$, the position vector of the parabola is

$$\mathbf{r}_0(x) = x\mathbf{e}_1 + f\left(1 - \frac{x^2}{l^2}\right)\mathbf{e}_2 \tag{1}$$