Learning and Individual Differences xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

Profiles of rational number knowledge in Finnish and Flemish students – A multigroup latent class analysis

Jake McMullen[a,*], Jo Van Hoof[b], Tine Degrande[b], Lieven Verschaffel[b], Wim Van Dooren[b]

[a] Department of Teacher Education and Centre for Research on Learning and Instruction, University of Turku, Finland
[b] Centre for Instructional Psychology and Technology, KU Leuven, Belgium

A R T I C L E I N F O

Keywords:
Rational numbers
Latent class analysis
Natural number bias
International comparison

A B S T R A C T

Students have a great deal of difficulties learning about rational number concepts, as they are confounded by misapplying reasoning about natural numbers to fractions and decimals, referred to as a natural number bias. For example, students often think that the number of digits of a decimal, or the size of the component numbers of fractions, is enough information to determine the magnitude of rational numbers. As well, students have trouble understanding that there is an infinite number of numbers between any two fractions or decimals. Using multigroup latent class analysis, the present study examines the structure of 611 Finnish and Flemish students' rational number knowledge in order to determine the similarities and differences between these two sub-samples. Results reveal that, while the Flemish students performed somewhat better, there were only relatively minor differences in the structure of the two sub-samples' rational number knowledge. In general, it appears that the natural number bias affects these Finnish and Flemish students' knowledge of the size and density of fractions and decimals in similar ways.

1. Introduction

Learners – in wide age ranges and across nationalities – face difficulties when learning about rational numbers (Torbeyns, Schneider, Xin, & Siegler, 2015; Vamvakoussi, Christou, Mertens, & Van Dooren, 2011). These difficulties are especially troubling given the importance of rational numbers in both later mathematical learning (DeWolf, Bassok, & Holyoak, 2015; Siegler et al., 2012) and work-life activities (Handel, 2016). Part of these difficulties are accounted for by difficulties in the transition from reasoning about numbers as being natural number exclusively, to numbers including also rational numbers (Ni & Zhou, 2005). While natural number knowledge can often be used to reason successfully about rational numbers, they also have some features that are different from natural numbers and thus an overreliance on natural number reasoning has been identified as a major cause of difficulties with rational number learning (Vamvakoussi, Van Dooren, & Verschaffel, 2012). This *natural number bias* has been identified in a large number of instances (e.g., Kainulainen, McMullen, & Lehtinen, 2017; Nunes & Bryant, 2008; Obersteiner & Tumpek, 2016; Vamvakoussi et al., 2011; Van Hoof, Janssen, Verschaffel, & Van Dooren, 2015). While the universality of such a bias across educational contexts has been examined with one particular feature of the bias, namely the density of the set of rational numbers (Vamvakoussi et al., 2011), this has never been done on a holistic level. The present study aims to examine, in distinct samples of Finnish and Flemish children, the extent to which there are similarities and differences in the overapplication of natural number concepts when reasoning about multiple aspects of rational numbers across the two sub-samples.

1.1. Rational number conceptual knowledge and the natural number bias

Many aspects of rational number knowledge can be extrapolated from natural numbers. For example, with fractions that have the same denominators, adding together the numerators will give you the correct numerator for the sum, and numbers with the same number of decimal places can be compared in the same way as natural numbers. Not only can natural number knowledge be a boon for understanding some parts of rational numbers; there are also many types of difficulties students face when reasoning and learning about rational numbers, in contrast to their natural number development. For example, the sheer complexity of fraction notation makes learning fraction procedures more challenging than learning procedures with natural numbers (Behr, Lesh, Post, & Silver, 1983). However, a large body of research suggests that a substantial part of the difficulties students face when learning about rational numbers stems from a natural number bias (e.g., Ni & Zhou, 2005).

* Corresponding author at: Department of Teacher Education, Turun Yliopisto, 20014, Finland.
E-mail address: jake.mcmullen@utu.fi (J. McMullen).

https://doi.org/10.1016/j.lindif.2018.02.005
Received 30 November 2016; Received in revised form 31 January 2018; Accepted 5 February 2018
1041-6080/

One of the first instances of this bias, is reasoning about the *size* of rational numbers (Kainulainen et al., 2017; Stafylidou & Vosniadou, 2004; Van Hoof, Janssen, et al., 2015). Numerous studies (e.g. Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Vamvakoussi et al., 2012; Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013) have found that even educated adults show a bias when reasoning about the size of fractions and decimals when their magnitudes are not congruent with their natural number features (e.g., while 6 is smaller than 7, 1/6 is larger than 1/7). Students face a number of particular difficulties with rational number size concepts. First, like educated adults, students often take larger numerators and denominators to represent larger fraction magnitudes (e.g., Moss, 2005). As well, students often believe that decimal numbers' magnitudes can be determined by the length of the decimal, often equating larger magnitudes with longer decimals (e.g., Durkin & Rittle-Johnson, 2015). Research has shown that many students still hold these conceptual misunderstandings even after multiple years of instruction (Van Hoof et al., 2013).

A stronger feature of the natural number bias is the difficulty students have in understanding the *dense nature* of the set of rational numbers (McMullen, Laakkonen, Hannula-Sormunen, & Lehtinen, 2015; Vamvakoussi & Vosniadou, 2004). As with size concepts (e.g. Kainulainen et al., 2017; Stafylidou & Vosniadou, 2004), there is evidence that students need to undergo radical conceptual change in order to gain a mathematically correct understanding of the set of rational numbers as being infinitely dense, with an infinite number of numbers – both fractions and decimals – between any two numbers, be them fractions or decimals (Merenluoto & Lehtinen, 2004; Vamvakoussi et al., 2011; Vamvakoussi & Vosniadou, 2010). For the most part students show a strong bias in favor of natural number concepts with the set of rational numbers, claiming that there are no, or a limited number of numbers between, for example, 1/5 and 2/5. Once again, educated adults, and even mathematical experts, exhibit signs of the natural number bias when reasoning about the density of rational numbers (Vamvakoussi et al., 2012). The natural number bias with density concepts is also extremely resilient to teaching, though it is rarely explored as an explicit topic in mathematics classrooms in primary or lower secondary schools (McMullen et al., 2015).

Evidence suggests that the natural number bias may be stronger for density than for size concepts (McMullen et al., 2015; Van Hoof, Vandewalle, Verschaffel, & Van Dooren, 2015), but that these two concepts may also be interconnected (Van Hoof, Janssen, et al., 2015). Students' struggles with understanding both size and density concepts may have the common underlying cause of the natural number bias and thus they can be considered unidimensional (ibid.). There is also evidence to suggest that understanding of size concepts is necessary, but not sufficient for coming to understand density concepts (McMullen et al., 2015). This suggests a common underlying natural number bias that accounts for features of students' difficulties for both size and density concepts. In addition, these previous studies have suggested that the natural number bias is unified across reasoning about both fractions and decimals, even though some students incorrectly consider these two representations of rational numbers to be separate (Vamvakoussi & Vosniadou, 2010). The present study aims to further examine the similarities and differences in the patterns of students' fraction and decimal size and density knowledge in both a sample of Finnish and a sample of Flemish students.

1.2. Conceptual change and latent variable mixture models

Previously, Vamvakoussi et al. (2011) compared Greek and Flemish secondary students' knowledge of density concepts. Results indicated that, in general, patterns of knowledge of density concepts were consistent across the two samples, though Flemish students performed better than the Greek students. These results suggest that the development of density knowledge is somewhat universal since radical conceptual change is needed in both Greek and Flemish students in order to have a mathematically correct concept of the density of the rational number set (see also Vamvakoussi & Vosniadou, 2010). However, the two-step cluster analysis used by the authors did not allow for a proper comparison of whether the patterns of knowledge between the two samples were similar. The two samples were included in the same cluster analysis and all students were assigned to one of the same five clusters based on this analysis. It is possible, however, that there were structural differences in the patterns of knowledge between the students from Flanders and Greece that were not captured in this analysis. While this analysis provided an overall view of the differences in density knowledge between students, it did not allow for the explicit comparison of the patterns of knowledge between the two sub-samples.

Recently, a number of studies have employed latent variable mixture models in order to examine possible qualitative changes in conceptual knowledge that would be predicted by theories of conceptual change (Edelsbrunner & Stern, this issue; Flaig et al., this issue; Kainulainen et al., 2017; Schneider & Hardy, 2013; Straatemeier, van der Maas, & Jansen, 2008). In particular, the radical restructuring of knowledge predicted by the framework theory of conceptual change (Vosniadou, 2014) has previously been examined through the use of latent variable mixture models, such as latent class and latent transition analyses (for an overview see Hickendorff et al., this issue). These types of analyses have been employed with concepts of sinking and floating (Edelsbrunner & Stern, this issue; Schneider & Hardy, 2013), astronomy (Straatemeier et al., 2008), rational number size (Kainulainen et al., 2017), and human memory processes (Flaig et al., this issue). In these cases, the claim is that there are not only quantitative differences in the amount of correct knowledge students have, but there may be also qualitative differences in the structure and types of knowledge students have, which are not typically captured using continuous or sum scores. The use of these types of latent variable mixture models allows researchers to capture among other things, intermediate states of understanding, which differ both from each other and from scientifically or mathematically correct concepts in qualitative ways.

For example, Schneider and Hardy (2013) found that intermediate levels of understanding of concepts of sinking and floating could be found that did not differ in the number of correct answers the students gave, but had qualitative differences in the patterns of responses that impacted their later development towards a scientifically correct concept. As well, Kainulainen et al. (2017) found that students' concepts of rational number size shifted from (1) coherent natural number dependent concepts, through (2) incoherent, intermediate phases, characterized by different synthetic concepts of number, in which natural and rational number concepts are used in inconsistent manners, and into (3) coherent, mathematically-correct concepts of rational number size. This study indicated that latent variable mixture models were useful for identifying the qualitative differences in students' rational number knowledge. These qualitative differences are especially relevant to distinguish between different intermediate phases in which overall performance as measured by a sum score may have indicated similar levels of knowledge, whereas Kainulainen and colleagues' findings suggest differing levels of later success across these intermediate classes. However, this study only examined knowledge of size concepts in a single educational context, through the use of sum scores, which cannot provide a detailed view on patterns of knowledge. A more complex method using latent variable mixture models allows for the direct comparison of the structure of response patterns across groups (Geiser, Lehmann, & Eid, 2006). For the first time, the present study will employ such a multigroup latent class analysis (LCA) in order to examine whether students have similar patterns of knowledge with rational number size and density concepts in both the sample of Finnish students and the sample of Flemish students participating in the present study.