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The crossing number of locally twisted cubes LTQ_n ☆

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ABSTRACT

The *crossing number* of a graph G is the minimum number of pairwise intersections of edges in a drawing of G. Motivated by the recent work (Faria et al., 2008) which solves the upper bound conjecture on the crossing number of n-dimensional hypercube proposed by Erdős and Guy, we consider the crossing number of locally twisted cubes LTQ_n, which is one of important variation of the hypercube Q_n. In this paper, we obtain the upper bound of the crossing number of LTQ_n as follows.

$$cr(LTQ_n) \leq \frac{87}{512}4^n - \frac{4n^2 - 15 + (-1)^{n-1}}{32}2^n.$$

1. Introduction

The *crossing number* $cr(G)$ of a graph G is the minimum number of pairwise intersections of edges in a drawing of G in the plane. The notion of crossing number is a central one for Topological Graph Theory and has been studied extensively by mathematicians including Erdős, Guy, Turán and Tutte, et al. (see [8,13,25,26,32,38,39,42]). In the past thirty years, it turned out that crossing number has many important applications in discrete and computational geometry (see [2,17,23,24,33,34,36,37]).

The immediate applications in VLSI theory and wiring layout problems (see [1,18,19,31]) also inspired the study of crossing number of some popular parallel network topologies such as hypercube and its variations. A suitable interconnection network is an important part for the design of a multicomputer or multiprocessor system. This network is usually modeled by a symmetric graph, where the nodes represent the processing elements and the edges represent the communication channels. Desirable properties of an interconnection network include symmetry, embedding capabilities, relatively small degree, small diameter, scalability, robustness, and efficient routing. The crossing number is an important parameter to measure embedding capabilities of interconnection network. Among all the popular parallel network topologies, hypercube is the first to be studied (see [5,6,9,10,22,35]). An n-dimensional hypercube Q_n is a graph in which the nodes can be one-to-one labeled with 0–1 binary sequences of length n, so that the labels of any two adjacent nodes differ in exactly one bit.

Computing the crossing number was proved to be NP-complete by Garey and Johnson [12]. Thus, it is not surprising that the exact crossing numbers are known for graphs of few families and that the arguments often strongly depend on their structures (see for example [11,20,27,30,45–50]). Even for hypercube, for a long time the only known result on the exact value of crossing number of Q_n has been $cr(Q_3) = 0$, $cr(Q_4) = 8$ [5], $cr(Q_5) \leq 56$ [22]. Hence, it is more practical to find

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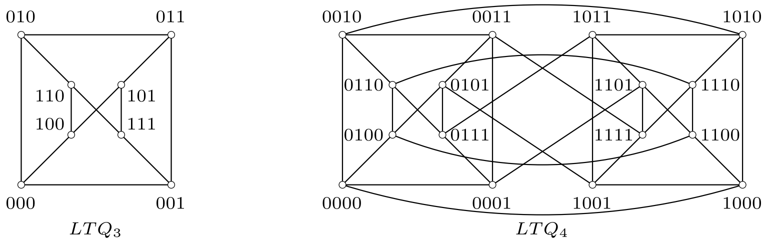

Fig. 1.1. Locally twisted cubes LTQ_3 and LTQ_4.

upper and lower bounds of crossing numbers of some kind of graphs. Concerned with upper bound of crossing number of hypercube, Erdős and Guy [8] in 1973 conjectured the following:

$$cr(Q_n) \leq \frac{5}{32}4^n - \lfloor \frac{n^2+1}{2} \rfloor 2^{n-2}.$$

In 2008, Faria, Figueiredo, Sykora and Vrt'o [10] constructed a drawing of Q_n in the plane which has the conjectured number of crossings mentioned above. Early in 1993 Sykora and Vrt'o [35] also proved a lower bound of $cr(Q_n)$:

$$cr(Q_n) > \frac{1}{20}4^n - (n^2+1)2^{n-1}.$$

Since the hypercube does not have the smallest possible diameter for its resources, to achieve smaller diameter with the same number of nodes and links as an n-dimensional cube, a variety of hypercube variants were proposed. Such as augmented cube, folded hypercube and locally twisted cube, they not only retain some favorable properties of Q_n but also possess some embedding properties that Q_n does not.

The *n-dimensional augmented cube* AQ_n proposed by S.A. Choudum and V. Sunitha [4] in 2002. It is defined recursively as follows.

(a) AQ_1 is a graph isomorphic to Q_1.

(b) For $n \geq 2$, AQ_n is built from two disjoint copies of AQ_{n-1} according to the following steps. Let $0AQ_{n-1}$ ($1AQ_{n-1}$) denote the graph obtained by prefixing the label of each vertex of one copy of AQ_{n-1} with 0(1) and connect each vertex $x = 0x_2x_3 \ldots x_n$ of $0AQ_{n-1}$ with the vertex $1x_2x_3 \ldots x_n$ of $1AQ_{n-1}$ by an edge and the vertex $1\overline{x_2}\overline{x_3} \ldots \overline{x_n}$ of $1AQ_{n-1}$ by an edge.

In 2013, Wang Guoqin, Wang Haoli and Yang Yuansheng et al. [40] constructed a drawing of AQ_n and shown $cr(AQ_n) < \frac{13}{16}4^n - \frac{4n^2+7n-12}{8}2^n$ for $n \geq 8$.

The *n-dimensional folded hypercube* FQ_n is a graph obtained from n-dimensional hypercube by adding all complementary edges. It was proposed by El-Amawy and Latifi [7] in 1991.

Recently, Wang Haoli and Yang Yuansheng et al. [41] constructed a drawing of FQ_n and shown $cr(FQ_n) \leq \frac{11}{32}4^n - \frac{n^2+3n}{8}2^n$ for $n \geq 3$.

Yang et al. [44] first proposed the locally twisted cubes LTQ_n in 2005 and proved that LTQ_n contains cycles of all lengths from 4 to 2^n. Ma and Xu [21] and Hu et al. [16], independently, improved this result by proving that $LTQ_n (n \geq 2)$ is edge-Pancyclic. Even when faulty elements occur, Chang et al. [3] and Park et al. [29], independently, showed that $LTQ_n (n \geq 3)$ is $n-2$ fault-tolerant Pancyclic. Xu et al. [43] showed that $LTQ_n (n \geq 3)$ is $n-3$ fault-tolerant edge-Pancyclic. Park et al. [28] showed that LTQ_n is $n-2$ fault-tolerant Hamiltonian and $n-3$ fault-tolerant Hamiltonian-connected. Hsieh et al. [14,15] showed that LTQ_n with at most $2n-5$ faulty edges contains a fault-free Hamiltonian cycle under the conditional-fault assumption and presented an algorithm for constructing Edge-Disjoint Spanning Trees (EDSTs) in LTQ_n. The locally twisted cube keeps as many nice properties of hypercube as possible and is conceptually closer to traditional hypercube, while it has diameters of about half of that of a hypercube of the same size. Therefore, it would be more attractive to study the crossing number of the n-dimensional locally twisted cubes.

The *n-dimensional locally twisted cube* LTQ_n is defined recursively as follows.

(a) LTQ_2 is a graph isomorphic to Q_2.

(b) For $n \geq 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} according to the following steps. Let $0LTQ_{n-1}$ ($1LTQ_{n-1}$) denote the graph obtained by prefixing the label of each vertex of one copy of LTQ_{n-1} with 0(1) and connect each vertex $x = 0x_2x_3 \ldots x_n$ of $0LTQ_{n-1}$ with the vertex $1(x_2 + x_n)x_3 \ldots x_n$ of $1LTQ_{n-1}$ by an edge, where $+$ represents the modulo 2 addition.

The graphs shown in Fig. 1.1 are LTQ_3 and LTQ_4, respectively.

In this paper, we studied the crossing number of the n-dimensional locally twisted cube LTQ_n, and obtained the upper bound of the crossing number of LTQ_n as follows.

$$cr(LTQ_n) \leq \frac{87}{512}4^n - \frac{4n^2 - 15 + (-1)^{n-1}}{32}2^n.$$

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