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Topology optimization for heat conduction by combining level set method and BESO method

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A B S T R A C T

The optimal design problem of heat conduction is solved by combining the level set method and the bi-directional evolutionary optimization (BESO) method. It is well known that the level set method is not able to nucleate holes in the structure during the optimization. In order to address this issue, the material removal scheme of the BESO is combined in the level set based topology optimization. In other words, hole is nucleated by using the material removal scheme. The criterion for hole nucleation is explained, and the optimization procedure is described. The results of numerical examples prove that the proposed method is effective and efficient.

1. Introduction

Efficient heat conduction is important for many engineering applications, and topology optimization for heat conduction has been studied by using several methods, including the homogenization method [1,2], the SIMP (Solid Isotropic Microstructure with Penalization) method [3–7], the ESO (evolutionary structural optimization) method [8–10], and the level set method [11–15]. The reader is referred to [16] for an excellent review.

In the present paper, the topology optimization for heat conduction is solved through an alternative method, i.e., by combining the level set method and the bi-directional evolutionary optimization (BESO) method. More specifically, the material removal scheme of BESO is used in the level set based topology optimization to nucleate holes.

In the level set method, structure and its boundary are directly presented and evolved. However, it is well known that the level set method cannot nucleate holes in the interior of a structure [17,18]. The topological changes allowed by the level set method only include vanish of a hole, merge of holes, and break apart of a region. In other words, the topology can only become simpler as the design evolves. This makes the level set based topology optimization, especially in 2D, depend on the initial design. In 3D the situation is slightly different. Although a new hole cannot be nucleated right in the interior of a 3D structure, a hole can be "tunneled" through the material in between two pieces of boundary [19].

Conventionally, the level set method is combined with the topological derivative [20–23] to nucleate holes during the optimization. The topological derivative is either periodically called in an independent step [12,24–26] or included into the Hamilton-Jacobi equation as an additional term [27,28]. Comparing the material removal scheme of the BESO and the topological derivative, one can see that when a very small amount of inefficient material is removed from the interior of a structure according to the BESO, the effect is essentially the same as that of using the topological derivative to nucleate a hole. In fact, people found out that the topological derivative is closely related to shape sensitivities [21] and density sensitivities [23,29,30]. However, the concept of BESO is simpler, and it is easier for numerical implementation, thus being more friendly to engineers.

Besides the combination of topological derivative with the level set method, some variants of the level set method were also proposed to let holes naturally appear during the evolution of structure. The radial basis function (RBF) based level set methods use RBF to parameterize the level set function [31–34], and several drawbacks of classical level set method including the inability to nucleate holes were overcame. Reaction–diffusion equation was used to update the level set function [35], and it allowed for nucleation of holes. A semi-implicit additive operator splitting (AOS) scheme was used to solve the Hamilton–Jacobi equation of the level set method [36], and it was capable of creating new holes.

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A secondary level set function that represents a pseudo third dimension in two-dimensional problems was used to facilitate new hole insertion [37]. These method are successful. In the present study, however, yet another alternative method is proposed, i.e., combining the level set method with the material removal scheme of the bi-directional evolutionary structural optimization (BESO).

The paper is organized as follows. In Section 2 the heat conduction topology optimization problem is described. In Section 3 the level set method and boundary variation are described. In Section 4 the BESO method and hole nucleation are described. Section 5 describes the optimization procedure. Section 6 gives numerical examples and discussions. Section 7 concludes this paper.

2. Optimization problem

A structure is represented as an open bounded set $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3). It is required that during optimization all admissible structures should stay within a fixed reference domain $D \subset \mathbb{R}^d$, i.e., $\Omega \subset D$. The boundary of Ω consists of three disjoint parts, i.e.,

$$\partial\Omega = \Gamma_N \cup \Gamma_H \cup \Gamma_D$$

where a Neumann boundary condition is imposed on Γ_N, a homogeneous Neumann boundary condition on Γ_H, and a Dirichlet boundary condition on Γ_D. In the present study, only Γ_H is subjected to optimization; Γ_N and Γ_D are fixed.

The strong form of heat conduction equation in the present study is given by

$$\begin{cases} -\kappa\nabla^2 T = b & \text{in } \Omega \\ \kappa\nabla T \cdot n = q & \text{on } \Gamma_N \\ \kappa\nabla T \cdot n = 0 & \text{on } \Gamma_H \\ T = 0 & \text{on } \Gamma_D \end{cases} \tag{1}$$

where κ is the isotropic thermal conductivity coefficient; T is the temperature; b is the rate of internal heat generation; q is the heat flux in the inward normal direction; n is the outward unit normal vector of the structure boundary. Heat convection is not considered in the present study.

The weak form of Eq. (1) is given by

$$a(T, \overline{T}) = \ell(\overline{T}), \quad \forall \overline{T} \in Y \tag{2}$$

where $a(T, \overline{T})$ and $\ell(\overline{T})$ are defined as

$$a(T, \overline{T}) = \int_\Omega \kappa\nabla T \cdot \nabla\overline{T} \mathrm{d}x \tag{3}$$

$$\ell(\overline{T}) = \int_\Omega b\overline{T} \mathrm{d}x + \int_{\Gamma_N} q\overline{T} \mathrm{d}s \tag{4}$$

and $Y$is the space of virtual temperature field given by

$$Y = \{\overline{T} \in H^1(\Omega)^d \mid \overline{T} = 0 \text{ on } \Gamma_D\} \tag{5}$$

The optimization problem is to minimize the thermal compliance given by

$$J = \ell(T) = \int_\Omega bT \mathrm{d}x + \int_{\Gamma_N} qT \mathrm{d}s \tag{6}$$

A constraint stating that the volume V of material should not be bigger than a given upper bound $\overline{V}$ is given by

$$V = \int_\Omega \mathrm{d}x \leqslant \overline{V} \tag{7}$$

The set of admissible shapes is defined as

$$\mathcal{U}_{\mathrm{ad}} = \{\Omega \subset D,\ V \leqslant \overline{V}\} \tag{8}$$

Now, the optimization problem is defined as

$$\inf_{\Omega \in \mathcal{U}_{\mathrm{ad}}} J \tag{9}$$

3. Level set method and boundary variation

3.1. level set method

Level set is a method to represent and track moving boundary, and more importantly, it is transparent to topological changes, which is significant for topology optimization. The level set method was first introduced to structural topology optimization by Sethian and Wiegmann [38], and it has caught much attention since the seminal papers [17,18,39,40].

According to the level set method, the boundary and inside/outside regions are defined as

$$\begin{aligned} &\Phi(x) = 0 \iff \forall\, x \in \Gamma_H \\ &\Phi(x) < 0 \iff \forall\, x \in \Omega \\ &\Phi(x) > 0 \iff \forall\, x \in D \setminus \overline{\Omega} \end{aligned}$$

The level set function Φ is specified by a regular sampling on a rectilinear grid and constructed to be a signed distance function to the boundary Γ_H. In such circumstances, the unit outward normal vector of the boundary can be readily obtained as by $n = \nabla\Phi$. Propagation of the boundary Γ_H is described by the Hamilton–Jacobi (H–J) equation

$$\frac{\partial\Phi}{\partial t} + V_n = 0 \tag{10}$$

A structured grid and the finite difference method (the first order upwind spatial differencing and forward Euler time differencing) are used to solve the H–J equation, and a reinitialization procedure is periodically performed.

In the present study, holes are inserted into a structure in an independent step during the optimization. When a hole, denoted as $B(p)$ where p is the center of the hole, needs to be inserted into the current design Ω^k, another level set function $\Psi(x)$ representing the hole is constructed, and the level set function $\Phi(x)$ is updated as

$$\Phi^{k+1}(x) = \max\{\Phi^k(x), \Psi(x)\} \tag{11}$$

Then, the updated structure becomes

$$\Omega^{k+1} = \Omega^k \setminus B(p) = \{x \mid \max\{\Phi(x), \Psi(x)\} < 0\} \tag{12}$$

When there exit more than one holes to be nucleated, the level set function $\Phi(x)$ will be repeatedly updated for each hole according to Eq. (11).

3.2. boundary variation

In level set based topology optimization, a proper variation of boundary is obtained as a descent direction in the admissible design space to improve the current design. To find such boundary variation, the material derivative and the adjoint method [17,18,41] are employed. Recently, the constrained variational principle was also used to find shape derivative of the Dirichlet boundary [42]. In the present study, the Dirichlet boundary is fixed and not subject to optimization. Therefore, the sensitivity analysis is conducted in the conventional way.

The Lagrangian of the optimization problem is defined as

$$\mathcal{L} = J(T) + a(T, \overline{T}) - \ell(\overline{T}) + \lambda(V - \overline{V}) \tag{13}$$

where $\overline{T}$ is the Lagrange multiplier for the heat conduction equation, and it is also called the adjoint temperature; λ is the Lagrange