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A numerical study of a liquid drop solidifying on a vertical cold wall

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ABSTRACT

Solidification of a liquid drop on a vertical wall is a typical phase change heat transfer problem that exists widely in natural and engineering situations. In this study, we present the fully resolved two-dimensional simulations of such a problem by a front-tracking/finite difference method. Because of gravity, the liquid drop assumed stick to the cold wall shifts to the bottom during solidification. Numerical results show that the conical tip at the solidified drop top is still available with the presence of volume expansion, but the tip location is shifted downward, resulting in an asymmetric drop after complete solidification. The tip shift, height and shape of the solidified drop are investigated under the influences of various parameters such as the Prandtl number *Pr*, the Stefan number *St*, the Bond number, the Ohnesorge number *Oh*, and the density ratio of the solid to liquid phases ρ_{sl}. We also consider the effects of the growth angle ϕ_{gr} (at the triple point) and the initial drop shape (in terms of the contact angle ϕ_0) on the solidification process. The most influent parameter is *Bo* whose increase in the range of 0.1–3.16 makes the drop more deformed with the tip shift linearly increasing with *Bo*. The tip shift also increases with an increase in ϕ_0. However, increasing *Oh* (from 0.001 to 0.316), *St* (from 0.01 to 1.0) or ρ_{sl} (from 0.8 to 1.2) leads to a decrease in the tip shift. Concerning time for completing solidification, an increase in *Bo*, ϕ_{gr} or ϕ_0, or a decrease in *St*, or ρ_{sl} results in an increase in the solidification time. The effects of these parameters on the drop height are also investigated.

1. Introduction

Solid–liquid phase change of drops is of crucial importance because the process widely exists in nature and engineering phenomena. For instance, water drops freeze on leaves [1], airplane [2], or wind turbine blades [3]. Metal drops solidify in deposition manufacturing [4] or in atomization [5]. Molten semiconductor drops crystallized during falling or on cold substrates have been used for solar cell applications [6–9]. Accordingly, understanding the complex heat transfer and solidification phenomena is extremely important to advance the above-mentioned technologies.

Concerning liquid drops solidifying on a horizontal plate, a vast number of works have been conducted. Experimentally, one can find this problem investigation in one of the pioneering works, Anderson et al. [10], in which the authors froze a water drop to demonstrate the accuracy of the proposed dynamic contact angle model at the tri-junction. An interesting feature observed from the experiment is the formation of an apex at the drop top after complete solidification. After then, a few experiments focusing on this singularity formed on frozen water drops have been done, e.g. [11–13]. Recently, this problem has been rapidly growing and getting much of attention. Jin et al. [14] froze a water drop sessile on a plate by lowering the plate temperature to a value below the water freezing point. The ice layer initially formed at the plate propagated to the drop top to complete the solidification. Similar works have been done in Zhang et al. [15] and Zhang et al. [16,17]. Focusing on not only water but also some other semiconductor materials (e.g. silicon, germanium and indium antimonide), Satunkin [18] reported the formation of conical solidified drops induced by volume expansion and growth angles at the tri-junction. Basing on theoretical analysis supported by the experiments, Satunkin found that the growth angle at the tri-junction is almost constant except near the end of the solidification process. Another work concerning the crystallization of a molten silicon drop can be found in Itoh et al. [9]. Theoretical studies on these problems can be found in [19–21], in which the authors mostly paid attention to the solidified drop profile and the temporal evolution of the solidification front assumed to be flat. Concerning numerical simulations of drops solidifying on a plate, a few studies have been performed by a boundary integral method [22], a finite element method [23], an enthalpy-based method [24], a volume of

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fluid method [17,25], and a front-tracking method in our recent works [26–29]. However, in these works, the authors considered only drops attached to a horizontal wall.

Related to drops solidifying on inclined surfaces, there have been many studies focusing on drops impacting, spreading and solidifying on the plates. For instance, Jin et al. [30,31] performed experiments for the freezing process of water on different inclined surfaces of different materials. To study the freezing process of asymmetric water drops, Ismail and Waghmare [32] tilted a cold plate and established the relationship between the asymmetry (in terms of contact angles) and the tip position and height of the frozen drops by scaling analysis and experiments. Wang and Matthys [33] experimentally investigated the heat transfer between the solidifying drops, of nikel and copper, and a metallic substrate tilted at an angle of 45°. Numerically, Zhang et al. [34] used a method of smoothed particle hydrodynamics to find the distribution of the temperature and movement of the solid–liquid interface of a drop impacting and solidifying on an inclined interface. Yao et al. [35] used the volume of fluid method implemented in an open source code OpenFOAM [36] to simulate the freezing process of a water drop. Some other numerical works can be found in [37,38].

Concerning drops on a vertical wall, Podgorski et al. [39] reported controlled experiments to show various shapes with remarkable temporal patterns of a water drop on a vertical plane. Smolka and SeGall [40] presented the formation of a fingering pattern, which tends to break up into drops, on the surface outside of a vertical cylinder. Li and Chen [41] experimentally formed frozen drops on a vertical wall and used the ultrasonic vibration to remove them from the wall. Numerically, Schwartz et al. [42] used a long-wave or lubrication approximation-based model to simulate three-dimensional unsteady motion of a drop on a vertical wall. Tilehboni et al. [43] used a lattice Boltzmann method to simulate a liquid drop moving on a vertical wall. However, the above-mentioned works have not considered solidification heat transfer.

Even though there have been many numerical studies on the phase change heat transfer of liquid drops, they mostly focused on the horizontal plate. The drop solidification on a vertical wall is rarely found. Accordingly, filling this gap is the main purpose of the present study because of its importance in academic and engineered applications [44–47]. We here use a two-dimensional front-tracking method, for tracking interfaces, combined with an interpolation technique, for dealing with the non-slip boundary on the solid surface, [26,48–50] to investigate the deformation with the solidification of a liquid drop attached to a vertical cold wall. Various parameters are investigated to reveal their effects on the solidification process.

2. Mathematical formulation and numerical method

We consider a liquid drop that attaches to a vertical wall kept at constant temperature T_c (Fig. 1). The liquid has a fusion temperature T_m higher than the temperature of the wall, $T_m > T_c$, and thus at the beginning, a thin liquid layer at the wall changes into solid. As time progresses, this solid layer evolves to the top of the drop. Here, we consider only a two-dimensional drop. We assume that the gas and liquid phases are incompressible, immiscible and Newtonian. In addition, the thermal (thermal conductivity k and heat capacity C_p) and fluid (density ρ and viscosity μ) properties are assumed constant in each phase. The one-fluid representation gives

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho u v}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} 2\mu \frac{\partial u}{\partial x} + \frac{\partial}{\partial y} \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \rho g_x + \vec{i} \cdot (\mathbf{F}_m + \mathbf{F}_\sigma) \tag{1}$$

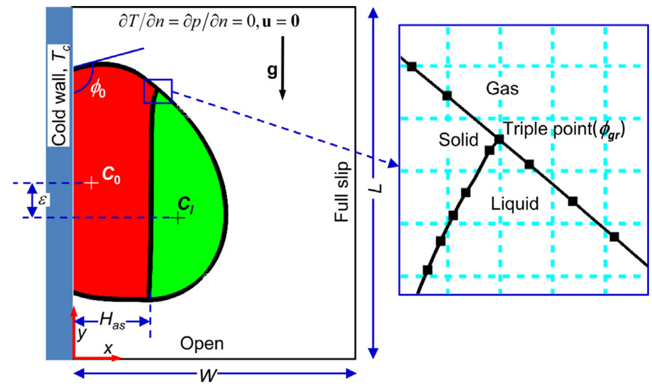

Fig. 1. A two-dimensional liquid drop, with an initial center of mass $C_0(x_{C0}, y_{C0})$, solidifies from a vertical cold wall. $C_l(x_{Cl}, y_{Cl})$ is the center of mass of the liquid part with $\varepsilon = y_{C0} - y_{Cl}$ called "shift" of the liquid part. H_{as} is the averaged height of the solidification front.

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho u v}{\partial x} + \frac{\partial \rho v^2}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} 2\mu \frac{\partial v}{\partial y} + \rho g_y [1 - \beta (T - T_m)] + \vec{j} \cdot (\mathbf{F}_m + \mathbf{F}_\sigma) \tag{2}$$

$$\frac{\partial(\rho C_p T)}{\partial t} + \frac{\partial(\rho C_p T u)}{\partial x} + \frac{\partial(\rho C_p T v)}{\partial y} = \frac{\partial}{\partial x} \left(\frac{k \partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{k \partial T}{\partial y} \right) + \int_f \dot{q}_f \delta(\mathbf{x} - \mathbf{x}_f) dS \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{4}$$

Here, $\mathbf{u} = (u, v)$ is the velocity vector. p is the pressure, $\mathbf{g} = (g_x, g_y)$ is the acceleration due to gravity. T is the temperature. β is the Boussinesq coefficient. f denotes interface. $\vec{i}$ and $\vec{j}$ are respectively the unit vectors on the x and y axes. $\mathbf{F}_m$ is the momentum forcing term used for enforcing the non-slip boundary condition on the solid surface [48,49]. $\mathbf{F}_\sigma$ is the interfacial tension force acting on the liquid–gas interface [51]:

$$\mathbf{F}_\sigma = \int_f \sigma \kappa \delta(\mathbf{x} - \mathbf{x}_f) \mathbf{n}_f dS \tag{5}$$

where σ is the interfacial tension coefficient, κ is the curvature, $\mathbf{n}_f$ is the unit vector normal to the interface. $\delta(\mathbf{x} - \mathbf{x}_f)$ is the Dirac delta function, which is zero everywhere except for a unit impulse at the interface $\mathbf{x}_f$. $\dot{q}$ is the heat flux at the solidification interface, given as

$$\dot{q} = k_s \frac{\partial T}{\partial n}\bigg)_s - k_l \frac{\partial T}{\partial n}\bigg)_l$$

with the subscripts s, l and g (when available) denoting solid, liquid and gas. In the present study, volume change induced by the density difference between the solid and liquid phases is also taken into account, and thus Eq. (4) is modified as [26]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{L_h} \left(\frac{1}{\rho_s} - \frac{1}{\rho_l} \right) \int_f \delta(\mathbf{x} - \mathbf{x}_f) \dot{q} dS \tag{6}$$

where L_h is the latent heat of fusion.

Initially, we assume the drop as a section of a sphere with a contact angle at the wall ϕ_0 and the apparent radius $R = [3V_0/(4\pi)]^{1/3}$, where V_0 is the initial volume of the drop. To simplify the problem, the temperature in the entire domain is set to T_0 equal to the fusion temperature, $T_0 = T_m$. In addition, a thin solid layer at T_c with a thickness of $0.02R$ is present at the plate at $t = 0$ [26]. The boundary conditions are shown in Fig. 1.