International Journal of Heat and Mass Transfer 127 (2018) 473–482

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Snowflake shaped high-conductivity inserts for heat transfer enhancement

H.C. Konan, E. Cetkin *

Izmir Institute of Technology, Department of Mechanical Engineering, Urla, Izmir 35430, Turkey

ARTICLE INFO

Article history:
Received 29 June 2018
Received in revised form 13 August 2018
Accepted 15 August 2018

Keywords:
Heat transfer enhancement
Fins
Constructal Theory
Snowflake

ABSTRACT

Here, we show numerically how thermal resistance in a two-dimensional domain with a point heat source can be reduced with embedded high-conductivity snowflake shaped pathways. The external shape of the domain is square, and its boundaries are heat sink. The geometry of the inserted pathways which corresponds to the minimum T_{max} was uncovered with the consideration of Constructal Theory, i.e. the constructal design. In the first assembly, number of mother (big) fins was uncovered as the area fraction increases. The results of the first assembly indicate that the increase in number of mother fins does not increase heat transfer after a limit number for the fins. After uncovering the mother pathway geometry corresponding to the minimum T_{max}, the daughter (small) fins inserted at the tip of them, i.e. second assembly. In the second assembly, the fin ratios, small fin location and angle were discovered when the area fraction is fixed. In addition, in the third assembly, larger daughter fins were attached to mother fins. The results of the second and third assemblies document what should be the geometric length scales and the number of daughter fins in order to minimize T_{max}. The constructal design uncovered is similar to the shape of snowflakes. Therefore, the results also uncover snowflakes correspond to the designs with minimum thermal conductivity, i.e., not mimicking the nature but understanding it with physics.

1. Introduction

While the size of electronic equipment decreasing, the number of components and their process capability in those devices are being increased. Therefore, dissipation of heat from the systems while keeping the temperature under a limit temperature becomes more challenging [1]. In order to satisfy the cooling requirement, miniaturization should be violated in convective cooling mechanisms as they require additional space for working fluid, piping and equipment. Whereas, no additional requirement exists for conductive heat transfer. Therefore, conductive heat transfer mechanism becomes more advantageous in miniaturized designs. Bejan [2] stated that conductive pathways are very influential to enhance heat transfer if they are optimized with the consideration of Constructal Theory.

Constructal law was stated by Adrian Bejan in 1996 as "For a finite-size flow system to persist in time (to live), its configuration must change in time such that it provides easier and easier access to its current." The Constructal law can be viewed as a guide to uncover the configurations of natural systems and to create new configurations for engineering systems [3]. Constructal law is applicable in many distinct fields from biology to transportation [4–7].

Furthermore, the literature documents that the thermal resistance of a domain can be minimized with insertion of high-conductivity pathways under the guidance of Constructal law. For instance, Bejan [2] indicated that the configuration of the pathways should be tree-shaped for minimum thermal resistance in a heat generating domain. Ledezma et al. [8] stated the number of branches and their thickness vary depending on the complexity of configuration during optimization. Almogbel and Bejan [9] also documented that there should be a spacing in between the low conductivity material and tip of the pathway in order to acquire minimum thermal resistance. Cetkin and Oliani [10] obtained asymmetric tree-shaped pathways with minimum thermal resistance for non-uniformly heat generated domain. They uncovered the design of the high-conductivity pathway is analogous to the design of roots where the water source is non-uniformly distributed. On the other hand, the literature also shows the design parameter optimizations for specific designs. Lorenzini et al. [11] documented the design of non-uniform X-shaped conductive pathways which correspond to minimum thermal resistance. Hajmohammadi et al. [12–14] documented the optimum fork-shaped, V-shaped and Phi-Psi shaped inserts. Horbach et al. [15]

* Corresponding author.
E-mail address: erdalcetkin@gmail.com (E. Cetkin).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.08.063
0017-9310/

Nomenclature

A	area [m^2]
k	thermal conductivity [$W\,m^{-1}\,K^{-1}$]
t_f	thickness of mother fin [m]
L_f	length of mother fin [m]
L	length of square [m]
m	number of mother fins
n	vector normal to solid-solid interface
T	temperature [K]
q''	heat flux [$W\,m^{-2}$]
x, y	spatial coordinates

Greek letters

ϕ	volume fraction
α	angle between small fins

Subscripts

b	big
h	high
f	fin
l	low
max	maximum
opt	optimized parameter
s	small

Superscripts

i	index of mesh independency test

documented the optimum design of Y-shaped conductive pathways. Feng et al. [16] uncovered the design parameters of optimum "+" shaped high-conductivity pathways to minimize thermal resistance. Eslami [17] documented the thermal performance of V-shaped pathway in a rectangular heat generating domain. You et al. [18] documented optimum configurations of constant and variable cross-sectional pathways in a non-uniform triangular heat generating domain. Lorenzini et al. [19–20] uncovered the effect of the thermal contact resistance on the optimization of I-shaped and T-shaped pathways. In addition, the various constructal fin shapes were documented in the literature [21–27].

Since 1951 scientists have classified snowflake shapes. First, they classified 10 different snowflake shapes [28]. Today, 121 classes exist for snow crystals and solid precipitation particles [29]. In addition, the current literature focuses on snowflake formation and creating new models in order to understand snowflake formation [30–31]. Furthermore, the current literature shows the usage of snowflake shapes to develop more efficient engineering systems [32–33]. Bejan et al. [34] uncovered analytically the solidifying materials should have spherical section in the center of the nucleation site with attached needle-like structure, i.e. snowflake shaped, because this design facilitates the flow of heat toward thermal equilibrium in a more effective manner.

Here, we document the how the fin shape should be altered to minimize thermal resistance in between a point source and a conductive domain. The volume of fins was fixed, and the thermal resistance was decreased with only varying the design under the guidance of Constructal law. The constructal design which is corresponding to the minimum thermal resistance is similar to a specific type of snowflake. We documented the number of mother and daughter fins, their locations, their dimensions, and angle in between the fins in the domain to obtain minimum T_{max}. Here, the design was not dictated as the design of a snowflake as literature focuses. The design is free to evolve, and it evolves to a snowflake design to minimize the resistance to its flow (i.e., heat).

2. Model

Consider a two-dimensional domain where heat is generated by a point heat source, the size of $\pi \times R^2$. The heat is generated with constant heat flux, q''. The external shape of the domain is a square with the scale of L. In the domain there are high-conductivity pathways, which are fins, in order to minimize T_{max}. The fins with high thermal conductivity k_h are rectangular which are embedded around the heat source. The rest of the domain is composed of low thermal conductivity material with low thermal conductivity k_l. The area of external shape (A_s) is fixed. Otherwise, fin area (A_f) and fin ratio (t_f/L_f) may change. The outer surface of the square domain with length scale of L the heat sink with 273 K. The other boundary condition is constant surface heat flux ($\partial T/\partial \theta = q''$) at the circle surface.

In addition to them, for the sake of simplicity heat transfer occurs in the steady state. The materials are isotropic with constant material properties. With this in mind, the heat transfer energy equations were solved simultaneously by using a finite element software, COMSOL Multiphysics 5.0 [35], respectively.

$$k_l\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0 \tag{1}$$

$$k_h\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0 \tag{2}$$

The heat flux continuity between high and the low conductivity material is,

$$k_l\left(\frac{\partial T}{\partial n}\right)=k_h\left(\frac{\partial T}{\partial n}\right) \tag{3}$$

where n is the normal vector to the interface of high and the low conductivity material.

3. Method

Consider radius of the point heat source is 0.01 m with 1 W/m^2 heat flux. The length of the square is 1 m, and the area of the square domain was fixed through the text, so is the area fraction of fins. Thicknesses and lengths were calculated depending on number of fins and volume fraction. Area fraction (ϕ) is the ratio of fin area divided by the area of square domain.

$$\phi=\frac{A_f}{A_s}=\frac{m\times L_f\times t_f}{L\times L} \tag{4}$$

where m is the number of fins.

Firstly, there are four symmetrical placed equilaterals spacing at the perimeter of the heat source of radius 0.01 m, while $t_f/L_f = 0.1$ and ϕ was varied in between 0.001 and 0.01.

Unstructured mesh with quadratic shape functions were used through the study. In order to uncover when the results become mesh independent, i.e. not a function of mesh size, mesh independency test was performed. The criterion is $\left|\left(T^i_{max}-T^{i+1}_{max}\right)/T^i_{max}\right| < 5\times10^{-6}$ for mesh dependency. Table 1 uncovers that the criterion was satisfied with the mesh number becomes greater than 4000