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Natural convection of a chemically reacting fluid in a concentric annulus filled with non-Darcy porous medium

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ABSTRACT

We investigate the natural convective flow in a reactor bounded by two concentric cylinders and filled with fluid-saturated porous media. The fluid is supposed to generate heat by an exothermic reaction. The problem is described using the non-Darcy model that includes the friction resulted from the macroscopic shear and the flow inertia. We solve the transformed equations using finite difference method. A detailed study of the flow and thermal fields and the heat transfer at the inner and outer walls of the annulus has been conducted for a wide range of the physical parameters, such as, the Darcy number, Forchheimer drag parameter, Frank-Kamenetskii number, Rayleigh number and outer radius of the annulus. Two counter-rotating vortices are generated in each half of the annulus. It is found from the variations of the streamlines and isotherms that for higher values of the physical parameters the outer vortices go upward and the maximum temperature is built up at the top of the annulus. However, for lower values of them the inner and outer vortices occur on the same line and the isotherms become nearly concentric. With the increase of the Darcy number and Rayleigh number, the maximum value of the magnitude of Nusselt number at the inner and outer walls increases whereas its minimum value decreases. As a result, the Nusselt number curves intersect each other. Besides the magnitude of Nusselt number at the inner and outer walls appreciably decreases due to the increase of the Frank-Kamenetskii number and outer radius of the annulus. These findings could be used for the advancement of the existing technology or the development of new technology, especially when an exothermic reaction plays a key role in the convection process in enclosed porous media.

1. Introduction

Natural convection caused by internal heat generation in enclosed porous media occurs in various mechanical, chemical and engineering applications such as nuclear reactors, underground disposal of radioactive waste materials, exothermic chemical reactions in packed bed reactors, storage systems of agricultural products [1], electrolytic processes, geothermal energy recovery [2], aircraft cabin design, ground water systems, buried cables, thermal energy recovery and thermal storage [3]. From technological point of view, the type of internal heat generation and the heat transfer characteristics through the bounding surfaces could play an important role in the convection process and thereby the system performance. The above-mentioned applications could be categorized by the nature of heating the enclosure either internally or externally. The first case arises in the presence of distributed internal heat sources or heat generating fluids. However, the external heating occurs owing to the different thermal conditions applied to the walls of the enclosure. It is worth mentioning that most of the past studies have concentrated on the natural convection subjected to the external heating [4]. In this study, the heat is presumed to be generated from an exothermic reaction in the fluid contained in the enclosed porous media.

Pop et al. have performed matched asymptotic analysis to examine the natural convection in an enclosed domain bounded by two horizontal concentric cylinders [5] and two concentric spheres [6]. In both cases, the domain was filled with fluid-saturated porous medium. Analytical solutions of stream function, tangential velocity, and temperature are valid for a short time but in the whole domain. It is observed that the flow field in the bounded region is divided into three regions such as an inner boundary layer adjacent to the inner surface, an outer boundary layer adjacent to the outer surface, and a core region between the two boundary layers. Caltagirone [7] theoretically and numerically studied the natural convection flow in an enclosed porous

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Nomenclature

A	reactant
a	concentration of the reactant
B	product of the reactant
C	Forchheimer drag parameter
Da	Darcy number
E	activation energy
F_k	Frank-Kamenetskii number
g	acceleration due to gravity
h	local heat transfer coefficient
K	permeability of the porous medium
k_0	pre-exponential factor
L_y	characteristic length of the rectangular domain
Nu	local Nusselt number
n	normal direction to the cylinder walls
Pr	Prandtl number
Q	heat of reaction
R	universal gas constant
Ra	Rayleigh number
r_o	outer radius of the annulus
t	time
T	temperature of the fluid
T_0	surrounding environment temperature
u, v	dimensionless velocity components in the x- and y-direction, respectively
$\mathbf{V}$	magnitude of the velocity vector, $(u^2 + v^2)^{1/2}$
x, y	dimensionless horizontal and vertical distances of the rectangular domain

Greek symbols

α	thermal diffusivity
β	coefficient of volumetric expansion
η	distance measured normal to the annulus
θ	dimensionless temperature
κ_f	thermal conductivity of the fluid
κ_T	thermal conductivity of the porous medium
λ	thermal conductivity
ν	kinematic viscosity of the fluid
ξ	distance measured streamwise direction
ρ	density of the fluid
τ	dimensionless time
ψ	stream function
ω	vorticity function

Subscripts

η	derivative with respect to η
ξ	derivative with respect to ξ

medium bounded by two horizontal concentric cylinders. Kuehn and Goldstein [8] conducted experiment and analyzed theoretically to provide a comprehensive understanding of flow properties and heat transfer characteristics of the natural convection in a horizontal annulus. Poulikakos [9] numerically studied the flow instability caused by the buoyancy force in a horizontal fluid layer extending over a porous substrate. It has been reported that a relatively simple composite geometry can change the buoyancy-driven flow characteristics. The onset of convection in an internally heated horizontal porous layer is theoretically examined by Gasser and Kazimi [10]. However, Facas and Farouk [11] numerically studied the transient and steady-state, two-dimensional natural convective flow in a porous concentric annulus heated from inside. Average Nusselt number variations with time and transient development of wall temperature have been illustrated for different Rayleigh numbers. Siddiqa et al. [12] have recently accomplished a work on the natural convective flow for two-phase dusty non-Newtonian fluid along a vertical surface. Results show how the flow and heat transfer are changed in the presence of dusty particles in the non-Newtonian fluid. Later, the similar problem has been analyzed by Siddiqa et al. [13] considering the nanofluid flow along a vertical wavy surface. It is revealed that the rate of heat transfer is considerably reduced with the increase of the diffusivity ratio parameter and the particle-density increment number.

Sekhar et al. [14] numerically examined the natural convection flow within an annulus bounded by two horizontal concentric cylinders and filled with a heat generating fluid. In addition, the effect of external heating has taken into account by imposing two distinct temperatures at the cylinder walls. Unicellular and bicellular flow patterns are recognized in each half cavity with the change of the ratio of generation of heat to the temperature differences of the cylinder surfaces. Vasseur et al. [2] numerically investigated the laminar convection flow in a porous medium in the presence of uniformly distributed heat sources. The porous medium is enclosed by two concentric cylinders which are kept at constant temperature. It is observed that two counter-rotating vortices are generated in each half cavity. The ratio of the radius of outer cylinder to the radius of inner cylinder significantly affects the comparative size and intensity of the vortices. For higher values of the Rayleigh number, there is seen considerably higher temperature in the upper part of the annulus. The heat transfer characteristics due to thermal convection in a porous annulus have been examined by Burns and Stewart [3]. The model includes the effect of internal heat generation on the convective heat transfer. Results have demonstrated a near wall boundary layer adjacent to the inner wall subjected to the flow injection into the porous media. Du and Bilgen [1] investigated the free convective heat transfer in a vertical porous cavity considering a uniform heat generation. The influences of Rayleigh number, Darcy number and aspect ratios on the streamlines, isotherms and heat fluxes have been presented.

A good work on the natural convection flow in a concentric annulus filled with porous-saturated media was accomplished by Kaviany [15]. The effects of inertia force, boundary conditions and velocity-square terms have been incorporated in the model. Numerical results clearly demonstrate the significant reduction of heat transfer caused by these effects. Using a flow regime diagram, the demarcation lines among the pseudo-conduction, Darcy and non-Darcy regimes are also presented. Wu et al. [16] have investigated the transient and steady natural convection in eccentric annuli filled with porous media which is saturated with temperature dependent viscous fluids. The problem has been formulated with Brinkman–Darcy–Forchheimer model which was solved numerically. The heat transfer characteristics are determined for a variety of the Raleigh number, the eccentricity, porosity, and Darcy number.

Very recently, the heat transfer within an eccentric annulus filled with heat generating fluid has been studied by Imtiaz and Mahfouz [4]. The inner cylinder was assumed to be heated and the outer cylinder was maintained at a constant temperature. Also, Roy [17] has formulated a model for a two-dimensional closed reactor bounded by a wavy wall. The internal heating of the fluid is due to the generation of heat from the exothermic reaction. Results reveal that a transition from the steady-state to the