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Technical Note

Notes on factitious shear work of slip flow in a channel

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ABSTRACT

Gas slip flow is observed in a micro scale channel whose characteristic length is less than about 10 μm under atmospheric conditions. To analyze the slip flow, the energy equation with the viscous dissipation term is solved with a boundary condition which includes a factitious sliding shear work term due to the slip at the wall to compensate the energy balance. Recently, an alternate method which uses the boundary condition without inclusion of the factitious sliding shear work term has been proposed. However, it seems that physics of the slip flow has not been properly understood by researchers. In this paper to clarify the issue, a very simple configuration, a steady state laminar slip flow in a hydro-dynamically fully developed region of a circular micro-tube with an adiabatic wall, is considered. Also all thermo-physical properties of the fluid are assumed to be constant. Theoretical justification to the boundary condition of the energy equation is provided using the discontinuity of the velocity of the slip flow on the wall.

1. Introduction

In 1958 Maslen [1] proposed a modification of the thermal boundary condition for an energy equation with the viscous dissipation term for a gaseous slip flow in a micro-channel. An extra term is added to the ordinary boundary condition to compensate a factitious sliding shear work. Since then, many researchers (e.g. Sparrow and Lin [2], Miyamoto et al. [3] and Ramadan [4]) solved the slip flow in a channel using the proposed modified boundary condition. Recently, Vocale et al. [5] proposed an alternate method which uses the ordinary boundary condition, to calculate the gaseous slip flow in a micro-channel. Although, Hong and Asako [6] provided a theoretical justification to Maslen's boundary condition, it seems that physics of this problem has not been properly understood by researchers. Therefore, the authors try to provide an alternate theoretical justification to this problem, including the recent proposed method by Vocale et al. [5].

2. Analysis

To clarify the problem a very simple configuration, such as a steady state laminar slip flow in a hydro-dynamically fully developed region of a circular micro-tube with an adiabatic wall is considered. Thermo-physical properties of the fluid including the density are assumed to be constant. A schematic diagram of the problem is depicted in Fig. 1. The velocity in the hydro-dynamically fully developed region of the slip flow is expressed as

$$u = u_c\left\{1 - \frac{r^2}{(d/2)^2}\right\} + u_s \tag{1}$$

where u_s is the slip velocity and u_c is the velocity difference between the velocity at the center and the slip velocity. The momentum equation is expressed as

$$0 = -\frac{dp}{dx} + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) \tag{2}$$

Substituting Eq. (1) into Eq. (2), we obtain

$$\frac{dp}{dx} = \frac{-16\mu}{d^2}u_c \tag{3}$$

The general form of the energy equation under no volume force can be found in any textbook as (e.g. [7,8]).

$$\rho\frac{D}{Dt}\left(h + \frac{1}{2}\vec{V}^2\right) = \vec{V}\cdot\vec{f} + \mu\phi + div(\lambda\, grad\, T) \tag{4}$$

where $\vec{f}$ is the internal force and expressed in the fully developed region as

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Nomenclature

d	tube diameter, m
f	Maxwell's reflection coefficient
$\vec{f}$	internal force per unit volume related to stresses, N/m^3
f_r, f_θ, f_x	force components, N/m^3
h	specific enthalpy, J/kg
Kn	Knudsen number
ℓ	mean free path, m
L	control volume length, m
$\dot{m}$	mass flow rate, kg/s
p	static pressure, Pa
$\dot{q}_w$	external wall heat flux, W/m^2
r, θ, x	cylindrical coordinates
T	temperature, K
t	time, s
u, v	velocity components, m/s
$\vec{V}$	velocity vector, m/s

Greek symbols

λ	thermal conductivity, W/(m·K)
μ	viscosity, Pa·s
ρ	density, kg/m^3
ϕ	dissipation function, m^2/s^2
τ	shear stress, N/m^2

Superscript

$-$	cross sectional average value

Subscript

c	center
s	slip flow

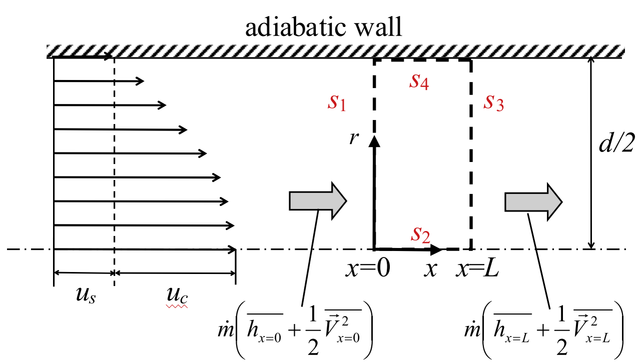

Fig. 1. Schematic diagram of a problem.

$$f_x = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}), \quad f_\theta = f_r = 0 \quad \text{and} \quad \tau_{rx} = \mu\frac{\partial u}{\partial r} \tag{5}$$

ϕ is the dissipation function and expressed in the fully developed region as

$$\phi = \left(\frac{\partial u}{\partial r}\right)^2 \tag{6}$$

Integrating Eq. (4) over the control volume shown by dashed lines in Fig. 1, then we obtain

$$\dot{m}\left(\overline{h_{x=L}}+\frac{1}{2}\overline{\vec{V}^2_{x=L}}\right) - \dot{m}\left(\overline{h_{x=0}}+\frac{1}{2}\overline{\vec{V}^2_{x=0}}\right) = \int_{C.V.}\left(\vec{V}\cdot\vec{f}+\mu\phi\right) r\,dr\,dx - \lambda\int_{s_1}\frac{\partial T}{\partial x} r\,dr + \lambda\int_{s_3}\frac{\partial T}{\partial x} r\,dr \tag{7}$$

Here, we assume no heat input from s_1 and s_3 boundaries. Then, Eq. (7) can be rewritten as

$$\dot{m}\left(\overline{h_{x=L}}+\frac{1}{2}\overline{\vec{V}^2_{x=L}}\right) - \dot{m}\left(\overline{h_{x=0}}+\frac{1}{2}\overline{\vec{V}^2_{x=0}}\right) = \int_{C.V.}(\vec{V}\cdot\vec{f}+\mu\phi)\, r\,dr\,dx \tag{8}$$

where $\dot{m}$ is the mass flow rate. $\overline{h_{x=0}}$, $\overline{h_{x=L}}$, $\overline{\vec{V}^2_{x=0}}$ and $\overline{\vec{V}^2_{x=L}}$ are the cross sectional average values at x = 0 and x = L and are obtained from the following equations:

$$\overline{h_{x=0}} = \frac{\int_0^{d/2}(\rho u h)_{x=0} r\,dr}{\dot{m}}, \quad \overline{h_{x=L}} = \frac{\int_0^{d/2}(\rho u h)_{x=L} r\,dr}{\dot{m}}$$

$$\overline{\vec{V}^2_{x=0}} = \frac{8}{d^2}\int_0^{d/2} u^2_{x=0} r\,dr, \quad \overline{\vec{V}^2_{x=L}} = \frac{8}{d^2}\int_0^{d/2} u^2_{x=L} r\,dr \tag{9}$$

Note that the depth of the control volume in θ direction is one radian.

Attention will turn to the RHS of Eq. (8). Since s_2 is the axisymmetric boundary, the RHS of Eq. (8) is expressed as

$$\int_{C.V.}(\vec{V}\cdot\vec{f}+\mu\phi)\,dV = -\int_{s_1} u\,\tau_{xx}\,r\,dr + \int_{s_3} u\,\tau_{xx}\,r\,dr + \int_{s_4} u_{r=d/2}\,\tau_{rx}\,(d/2)\,dx \tag{10}$$

Since τ_{xx} is zero on s_1 and s_3 in the fully developed region, Eq. (10) becomes

$$\int_{C.V.}\left(\vec{V}\cdot\vec{f}+\mu\phi\right)dV = \int_{s_4} u_{r=d/2}\,\tau_{rx}\,(d/2)\,dx = \mu\,u_{r=d/2}\left(\frac{\partial u}{\partial r}\right)_{r=d/2}(d/2)\,L \tag{11}$$

Then, Eq. (8) can be rewritten as

$$\dot{m}\left(\overline{h_{x=L}}+\frac{1}{2}\overline{\vec{V}^2_{x=L}}\right) - \dot{m}\left(\overline{h_{x=0}}+\frac{1}{2}\overline{\vec{V}^2_{x=0}}\right) = \mu\,u_{r=d/2}\left(\frac{\partial u}{\partial r}\right)_{r=d/2}(d/2)\,L \tag{12}$$

Let's consider three flow situations. The first one is a no slip flow in a stationary circular tube. In this case, the flow velocity at the center is u_c and the velocity on the wall is $u_s = 0$. The second one is no slip flow in a circular tube whose wall is "moving" in the flow direction with velocity, u_s. The velocity at the center is $u_c + u_s$. The last one is a slip flow in a stationary circular tube with the slip velocity, u_s. The velocity profile of these three cases is expressed by Eq. (1). Note that the pressure gradients of these three cases are identical since wall shear forces are identical. Also Eq. (12) which is the integral form of the energy equation, is available for these three cases. In the case of no slip flow in a stationary circular tube, the velocity on the tube wall is $u_{r=d/2} = 0$. Then, Eq. (12) becomes