Journal of Economic Behavior and Organization 154 (2018) 24–32

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

Subjective ambiguity and preference for flexibility

Leandro Gorno[a], Paulo Natenzon[b,*]

[a] FGV EPGE Escola Brasileira de Economia e Finanças, Rio de Janeiro, Brazil
[b] Department of Economics, Washington University in St. Louis, USA

ARTICLE INFO

Article history:
Received 10 July 2018
Revised 17 July 2018
Accepted 18 July 2018

Keywords:
Preference for flexibility
Pessimism
Subjective ambiguity
Self-paternalism

ABSTRACT

A preference over menus is monotonic when every menu is at least as good as any of its subsets. We show that every utility representation for a monotonic preference is equal to the minmax value of a decision maker whose payoff depends on the option chosen from the menu and on the realization of a subjective state. This representation suggests a decision maker who faces uncertainty about her own future tastes and who exhibits an extreme form of pessimism with respect to this uncertainty. In the case of finitely many alternatives, we provide a characterization of monotonic preferences which relaxes the submodularity axiom of Kreps (1979). We characterize the minimal state space needed for our representation, and we show that the second-period choice behavior of our decision maker differs from the one implied by the costly contemplation model of Ergin (2003).

1. Introduction

This paper analyzes preferences over menus of options. A preference over menus is *monotonic* when every menu is at least as good as any of its subsets. Theorem 1 shows that every utility representation for a monotonic preference is equal to the minmax value of a decision maker whose payoff depends on the option chosen from the menu and on the realization of a subjective state. This representation suggests a decision maker who faces uncertainty about her own future tastes and who exhibits an extreme form of pessimism with respect to this uncertainty.

We explore the consequences of Theorem 1 in the special case of a finite set of choice objects. This is the setting of the preference for flexibility model of Kreps (1979). Our *pessimism utility* representation result generalizes Kreps' model by relaxing the submodularity axiom. We provide a characterization of the minimal number of states necessary to write our representation.

The costly contemplation model of Ergin (2003) also generalizes Kreps' model in the finite setting. While the patterns of first-period choice among menus in our model are equivalent to those of Ergin's model, we show that costly contemplation and pessimism produce different predictions for second-period choice from menus.

2. Preference for flexibility

Let Z be an arbitrary set of choice objects and let $X \subset 2^Z \setminus \{\varnothing\}$ be a collection of non-empty menus containing Z. A preference over menus is given by a binary relation $\succcurlyeq \subset X \times X$. A preference over menus is *monotonic* if $x \supset x'$ implies $x \succcurlyeq x'$. A function $U : X \to \mathbb{R}$ is a utility representation for $\succcurlyeq$ whenever for all $x, x' \in X$ we have $x \succcurlyeq x'$ if and only if $U(x) \geq U(x')$. Our

* Corresponding author.
E-mail addresses: leandro.gorno@fgv.br (L. Gorno), pnatenzon@wustl.edu (P. Natenzon).

https://doi.org/10.1016/j.jebo.2018.07.009
0167-2681/

main result is that any utility representation for a monotonic preference is equal to the minmax value of a decision maker whose payoff depends on the option chosen from the menu and on the realization of a subjective state.

Theorem 1. *Every representation U for a monotonic preference $\succcurlyeq$ can be written as*

$$U(x) = \min_{s \in S} \max_{z \in x} u(z, s) \tag{1}$$

where S is a set of subjective states and $u : Z \times S \to \mathbb{R}$ is a state-dependent utility.

Proof. Since U represents $\succcurlyeq$ it must be bounded above, attaining its maximum at Z, the largest menu. Assume without loss of generality that $U(Z) = 0$.

Take $S = X$ and for each menu $s \in S$ let $\mathbb{I}_s : Z \to \mathbb{R}$ be the indicator function defined by $\mathbb{I}_s(z) = 1$ if $z \in s$ and $\mathbb{I}_s(z) = 0$ otherwise. Define $u : Z \times S \to \mathbb{R}$ by $u(z, s) = U(s)\mathbb{I}_s(z)$ for each $z \in Z$ and $s \in S$.

Note that for any $s \in S$ we have $\max_{z \in x} u(z, s)$ equal to $U(s)$ whenever $x \subset s$ and equal to zero otherwise. Also whenever $s \supset x$ we have $s \succcurlyeq x$ and therefore $U(s) \geq U(x)$. Moreover, for each $x \in X$ the set $\{\max_{z \in x} u(z, s) \colon s \in S\}$ attains its minimum at $s = x$. Hence the right-hand side of (1) is a well-defined function of x and for all $x \in X$ we have

$$\begin{aligned} \min_{s \in S} \max_{z \in x} u(z, s) &= \min_{s \supset x} \max_{z \in x} u(z, s) \\ &= \min_{s \supset x} U(s) \\ &= U(x). \end{aligned}$$

□

The representation (1) has the following interpretation. The decision maker chooses a menu today as if she were unsure about what her ranking of the alternatives in Z will be tomorrow. She foresees the possibilities represented by the state space S. Each state $s \in S$ yields a utility index $z \mapsto u(z, s)$ representing a possible ranking over the objects in Z. She is extremely pessimistic when evaluating which subjective state will occur, assessing each menu according to the worst-case scenario.

A parallel can be drawn between representation (1) and the multiple prior model of Gilboa and Schmeidler (1989). To see this formally, identify the choice of a menu $x \in X$ with an act f_x: $S \to Z \times S$ that delivers a payoff $f_x(s) = (\arg\max_{z \in x} u(z, s), s)$ which depends on the realization of the uncertain state $s \in S$. The decision maker exhibits an extreme form of ambiguity aversion and has a set of priors equal to the entire simplex $\Delta(S)$. She evaluates each menu $x \in X$ (represented by the act f_x) according to

$$\begin{aligned} U(x) &= \min_{s \in S} \max_{z \in x} u(z, s) \\ &= \min_{\mu \in \Delta(S)} \sum_{s \in S} \mu(s) \max_{z \in x} u(z, s) \\ &= \min_{\mu \in \Delta(S)} \int_S u \circ f_x \, d\mu \end{aligned}$$

which is the well-known functional form in the multiple prior model.

However, our model differs from Gilboa and Schmeidler's in two fundamental ways. First, the choice objects in the multiple prior model are acts, which map a set of states to a set of consequences, while our choice objects are menus of options. Second, in the multiple prior model the set of states is a primitive. In contrast, the state space in Theorem 1 is subjective, unobserved and obtained as part of the representation.

Remark 1. Theorem 1 is silent on the existence of a utility representation. These conditions are well-known. In particular, it follows immediately from Theorem 1 above and Theorem 3.1 in Fishburn (1970), that a preference relation $\succcurlyeq \subset X \times X$ admits a utility representation $U : X \to \mathbb{R}$ given by (1) if and only if it is complete, transitive, monotonic, and has an order-dense countable subset.

3. Finite environments

In this section, we explore the consequences of Theorem 1 when the set Z of choice objects is finite. This is the setting of the preference for flexibility model of Kreps (1979) and the costly contemplation model of Ergin (2003). Let $X = 2^Z \setminus \{\varnothing\}$ be the collection of all non-empty menus of objects from Z. Kreps (1979) imposes the following axioms on the preference $\succcurlyeq \subset X \times X$:

Axiom (A1)**.** Weak order: $\succcurlyeq$ is complete and transitive.

Axiom (A2)**.** Monotonicity: $x \supset x'$ implies $x \succcurlyeq x'$.

Axiom (A3)**.** Submodularity: $x \sim x \cup x'$ implies $x \cup x'' \sim x \cup x' \cup x''$ for all $x'' \in X$.

A1 is standard and allows a utility representation in this finite setting. A2 means the decision maker never values commitment, yet allows preference for flexibility. A3 says that if the decision maker does not value the flexibility provided by