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Expected efficiency based on directional distance function in data envelopment analysis

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ABSTRACT

Directional distance function (DDF), an evaluation technique that estimates relative efficiency of a decision making unit (DMU) along a pre-determined direction vector that is not restricted by the radial direction, has been widespread in productive efficiency research over the past two decades. A key challenge in DDF applications, however, is to decide on an appropriate (or the best) direction along which to measure efficiency. To circumvent this issue, we build on the DDF model and propose *expected efficiency* in efficiency estimation. Expected efficiency is defined as the mean value of all relative efficiency scores of a DMU along all directions. When calculating the overall relative efficiency score of a DMU, the expected efficiency model incorporates all possible directions rather than choosing a particular direction. As such, the expected efficiency approach extends DDF from a single direction to all directions. Some benefits of the expected efficiency approach include (1) relieving a decision maker of the burden of determining a particular directional vector among many choices; (2) overcoming a decision maker's subjectivity in the direction selection; (3) resolving the sensitivity issue caused by choosing different directions; and (4) ensuring that all DMUs are estimated in a consistent and equitable manner. Our study contributes to productive efficiency research and data envelopment analysis by introducing a new efficiency estimate that does not need to rely on one specific direction. Using two examples, we demonstrate the validity and the robustness of expected efficiency as an alternative efficiency estimate.

1. Introduction

Efficiency evaluation is integral to effective business and operations management. Research on the measurement of productive efficiency has advanced after the seminal work by Farrell (1957). Among various efficiency evaluation methods, data envelopment analysis (DEA) is one of the most important tools and has been adopted for performance evaluation in the areas of operations management, economics, public affairs, finance, etc. (Liu, Lu, Lu, & Lin, 2013; Emrouznejad & Yang, 2018). First introduced by Charnes, Cooper, and Rhodes (1978), DEA is a nonparametric linear programming method that measures the relative efficiencies of a set of comparable entities called decision making units (DMUs) with multiple inputs and multiple outputs (Cook & Seiford, 2009; Cooper, Seiford, & Zhu, 2011). In traditional DEA models such as the CCR (Charnes et al., 1978) and BCC (Banker, Charnes, & Cooper, 1984) models, each DMU chooses its own weights, that is, the radial direction to the origin, to obtain the optimal efficiency score. Restricted by the radial direction, traditional DEA models have two shortcomings. First, because each DMU follows its own radial direction in estimation, DMUs are not evaluated on the same basis. Thus, evaluation results vary and rankings are largely inconsistent (Sun, Wu, & Guo, 2013). Second, because a set of weights that is favorable to one DMU is not necessarily favorable to other DMUs, one DMU may dominate other DMUs (Kao & Hung, 2005; Wang, Chin, & Leung, 2009) thus the evaluation results may be unacceptable to other DMUs (Amin & Toloo, 2007; Wu, Chu, Sun, Zhu, & Liang, 2016).

To avoid the restriction of the radial direction, Chambers, Chung, and Färe (1996) extended the DEA models to other non-radial directions. Building on the distance function proposed by Shephard (1970) and Luenberger (1992). Chambers et al. (1996) proposed the directional distance function (DDF) to calculate relative efficiency of DMUs along a predetermined direction. Using DDF, a decision maker now has the flexibility in choosing either the same directional vector for all DMUs or a specific vector for each DMU (Aparicio, Pastor, & Vidal,

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2016). Once a directional vector is determined, the relative efficiency of each DMU can be obtained. Because the DDF approach can help DMUs to explore potential efficiency improvement along different directions, it has attracted considerable attention from researchers (e.g., Zhang & Choi, 2014; Toloo, Allahyar, & Hančlová, 2018). Accordingly, choosing an appropriate direction has become an area of focus in DDF-based modelling and applications (Krüger, 2016; Wang, Xian, Lee, Wei, & Huang, 2017).

Quite a few direction-selecting methods for DDF models, such as Simar, Vanhems, and Wilson (2012), Halkos and Tzeremes (2013), and Hampf and Krüger (2014), have been proposed in recent studies. These methods, however, estimate the efficiency of DMUs along a pre-determined direction that is subject to the decision maker's preference in selecting a direction, which leads to the potential subjectivity and reasonability concerns about the evaluation results (Wang et al., 2017). For example, Zhu (1996) discussed two situations in which unreasonable or unacceptable results may occur: (1) when the efficient targets are disliked by the decision maker, and (2) when these targets might not be attainable or realistic due to restricted managerial capabilities or other external factors.

Prior research has confirmed that the arbitrary selection of a direction would greatly influence evaluation results, including technical efficiency and scale efficiency (Vardanyan & Noh, 2006), along with efficiency change, technical change, and productivity change (Agee, Atkinson, & Crocker, 2012). Moreover, the efficiency evaluation based on a single direction, whether radial or non-radial, will always favor some DMUs while disfavoring other DMUs. Because evaluation results are sensitive to the direction chosen (Peyrache & Daraio, 2012), different direction-selecting methods have been proposed in prior studies (e.g. Asmild, Hougaard, Kronborg, & Kvist, 2003; Baek & Lee, 2009; Färe, Grosskopf, & Whittaker, 2013); in practice, this causes confusion for and adds burden to decision makers in choosing an appropriate (or the best) direction.

To address the above problems associated with selecting the direction in efficiency evaluation, this study proposes an alternative efficiency estimate, named expected efficiency, to circumvent the issue. We define expected efficiency as the mean value of all relative efficiency scores of a DMU along all directions. Our expected efficiency model incorporates the concept of mean value in the DDF model and takes into account all possible directions to the frontier in estimating a DMU's overall relative efficiency score. In mathematics, the mean and expected values are used interchangeably for sufficiently large datasets. Such an approach is inclusive and unbiased because the estimate includes all possible directions. Thus, expected efficiency is defined as the mean value of the relative efficiency scores of a DMU in all possible directions. A study by Peyrache and Daraio (2012) also seeks to tackle the direction selection issues in DDF. Their approach, however, includes only the directions towards the interior of the dominance area, which may neglect other directions that are not in the dominance set, and aggregates the directional measures by weights, which means each direction is not treated equally. In contrast, our expected efficiency approach includes all possible directions to the frontier and treats each direction equally.

Compared to the traditional DDF models, our proposed expected efficiency model has advantages of relieving the decision maker of the burden of determining the appropriate (or best) directional vector among many possible choices, especially when no specific direction can be justified. Instead of choosing an arbitrary directional vector or making a subjective (but perhaps biased) pick, the decision maker may be better off incorporating all directions to ensure equity and objectivity in the estimation. As such, we argue that expected efficiency can be adopted in practice as an alternative efficiency estimate.

The rest of the paper is organized as follows. Section 2 proposes the expected efficiency model based on DDF, along with the definition of expected efficiency. In Section 3, we introduce the numerical calculation method to calculate expected efficiency scores. Two examples are shown in Section 4. Finally, we conclude with the potential contributions of the expected efficiency-based DEA model in Section 5.

2. The expected efficiency model

Traditional DEA models (e.g. CCR by Charnes et al., 1978 and BCC by Banker et al., 1984), measure technical efficiency by the radial distance to the frontier. The intersection point of the envelopment frontier and the radius linking a particular DMU and the origin is called the frontier projection of that DMU. In input-oriented DEA calculations, the DEA efficiency is equal to the ratio of the distance from a DMU to its frontier projection over the distance from the DMU to the origin. In such a radial scenario, all inputs must decrease by the same proportion.

Charnes, Cooper, Golany, Seiford, and Stutz (1985) state that non-radial improvement may be possible, and DDF helps to explore various improvement directions not limited to the radial direction to the origin (Chambers et al., 1996; Chung, Färe, & Grosskopf, 1997; Chambers, Chung, & Färe, 1998). A specified DDF not only decides the promotion direction but also determines the improvement gap (Chambers et al., 1996). With the evaluated DMU projecting along different directions, the corresponding projection destination moves along the frontier. If a directional vector is determined, the DMU is compared with the corresponding projection, thus obtaining the relative efficiency under the directional vector. Fig. 1 presents different improvement directions for the DMU_E, where $\overrightarrow{EO}$ is the radial direction while others are not. Fig. 1 only considers the input-oriented improvement, and all of the improvement directions point to the bottom left side.

2.1. The definition of the directional distance function

We deal with n DMUs with the input and output matrices $X = (x_{ij}) \in \mathscr{R}^{m\times n}$ and $Y = (y_{rj}) \in \mathscr{R}^{s\times n}$, respectively. The production possibility set (PPS) T under the assumption of variable returns to scale (VRS, Banker et al., 1984) is defined below:

$$T = \left\{ (x_i, y_r) \left| \begin{array}{ll} \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_i, & i = 1, \cdots, m \\ \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant y_r, & r = 1, \cdots, s \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geqslant 0, & j = 1, \cdots, n \end{array} \right. \right\} \tag{1}$$

With respect to the directional vector $g = (-g_x, g_y) \neq 0_{m+s}$, $g_x \in R_+^m$, $g_y \in R_+^s$, the definition of the directional distance function (DDF) is (Chambers et al., 1998):

$$\overrightarrow{D_T}(x, y;g) = \sup\{\beta\colon\ (x-\beta g_x, y + \beta g_y) \in T\} \tag{2}$$

For the evaluated DMU_o ($o = 1, \cdots, n$), the relevant VRS linear programming formulation of the DDF can be written as shown in (3) (Aparicio et al., 2016):

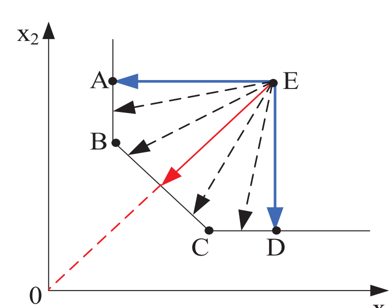

Fig. 1. The input-oriented projections of DMU_E.