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Violation of the Bell inequality in quantum critical random spin-1/2 chains

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ABSTRACT

We investigate the entanglement and nonlocality properties of two random XX spin-1/2 critical chains, in order to better understand the role of breaking translational invariance to achieve nonlocal states in critical systems. We show that breaking translational invariance is a necessary but not sufficient condition for nonlocality, as the random chains remain in a local ground state up to a small degree of randomness. Furthermore, we demonstrate that the random dimer model does not have the same nonlocality properties of the translationally invariant chain, even though they share the same universality class for a certain range of randomness.

1. Introduction

The use of quantum information tools in condensed matter systems has become widespread, mostly because of their usefulness for a better understanding of the behavior of quantum critical ground states (for a review, see Ref. [1]). Currently, entanglement and nonlocality measurements are under intensive scrutiny since they have shown to be able to signal quantum phase transitions[1] (QPTs) in many-body systems [2–10]. Even though these concepts are frequently associated with each other, it has been shown that they are indeed distinct by the construction of entangled mixed states which do not violate Bell-like inequalities[2] [11]. In addition, finding nonlocal states in many-body systems is of major interest, bearing in mind the many interesting applications of nonlocal states, such as to cryptography [12] and to the generation of random numbers [13].

Although it was observed that nonlocality measures may point out QPTs, it is far from clear what is the relation between nonlocality and QPTs. For instance, a recent study [14] has shown that due to monogamy and translational invariance, any mixed state of a spin pair of the critical XXZ spin-1/2 chain is a local state, i.e., any spin pair does not violate the Bell inequality (even though they can be in an entangled mixed state). This conclusion led us to inquire whether, generically, a critical state is always local.

Therefore, we consider here two different spin-1/2 chains with randomly generated coupling constants. By introducing randomness, we are able to break translational invariance without driving the system out of criticality. In one these random models, the critical state belongs to the so-called infinite-randomness universality class [15]. In this case, when the degree of inhomogeneities is very large, there are spin pairs in nearly Bell-like (singlet) states [16] which become strong candidates to violate the Bell inequality. In the other model, the corresponding universality class is of finite-disorder type. It was shown that the corresponding ground state has many similarities with the one of the translationally invariant case, such as sharing the same set of critical exponets (i.e., belonging to the same universality class) below a certain degree of randomness [17,18]. It is then much less clear whether the Bell inequality is violated or not.

We have shown here that the Bell inequality is violated in both cases, if the degree of randomness is greater than a certain amount (which we have determined). Moreover, for the case in the infinite-randomness universality class, the spin pairs violating the Bell inequality can be widely separated, while for the finite-disorder case only nearest-neighbor spin pairs may be in nonlocal states. The most striking result is that the second model exhibits

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[1] This is understood as a consequence of entanglement and nonlocality (as well as discord) inheriting the nonanalytic behavior at the critical point from the usual spin-spin correlation functions.

[2] When we refer to Bell inequalities or Bell-like inequalities we have in mind the original Bell inequality and the CHSH inequality (see Sec. 3).

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nonlocality even when it belongs to the same universality class of the translationally invariant case (which was shown to be local).

The remainder of this paper is structured as follows: in Sec. 2 we present our random models, emphasizing the differences between them. In Sec. 3 we define and describe how to obtain the entanglement and nonlocality measurements. Sec. 4 presents our numerical results, which are further discussed in Sec. 5 and followed by perspectives of future studies and applications.

2. The random uncorrelated and correlated XX spin-1/2 models

Here, we introduce the two studied models, which are special cases of the disordered XXZ spin-1/2 chain [19,20]. This model is described by the Hamiltonian

$$H = \sum_{i=1}^{L} J_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta_i S_i^z S_{i+1}^z \right), \tag{1}$$

where S_i^α are the usual spin-1/2 operators, $J_i > 0$ are the coupling constants, Δ_i are the anisotropy parameters and L is the chain size which we will assume to be even. In addition, we will consider periodic boundary conditions: $\mathbf{S}_{i+L} = \mathbf{S}_i$.

In the translationally symmetric case ($J_i = J$ and $\Delta_i = \Delta$) the system is critical for $-1 \leq \Delta \leq 1$ and it is described as an exotic Tomonoga-Luttinger spin liquid state [21], which is a highly entangled [1] but local state, i.e., any spin pair does not violate the Bell inequality [3,14].

Conversely, in the uncorrelated random case (J_i and Δ_i being uncorrelated and identically distributed random variables) the system is described as a critical random singlet state for $-1/2 < \Delta_i \leq 1$ [15,22] in which spin pairs can be highly entangled in nearly singlet states [16,23–25], as depicted in Fig. 1. Remarkably, it was shown that this state is universal, in the sense that all of its low-energy critical properties do not depend on (i) the details of the random variables, provided that the width of their distribution is not zero and not unphysically large, and on (ii) the system anisotropy, provided that $-1/2 < \Delta_i \leq 1$.

For this reason, we here restrict our study to the case known as the XX model, in which $\Delta_i = 0$, $\forall i$. Another reason for our choice is due to the existence of a mapping between the XX chain and the tight-biding model of free spinless fermions [26], which allows us to study considerably large chains via the exact diagonalization of the Hamiltonian (1). Finally, it is plausible that our conclusions for the XX model also extend to the XXZ model in the critical random-singlet region $-1/2 < \Delta_i \leq 1$ because, in this region, the ground state of the random XXZ chain depends very weakly on the values of the local anisotropies Δ_i, thus exhibiting the symmetry properties of the SU(2) symmetric Heisenberg model $\Delta_i = 1$ [27].

In our study, we draw the random couplings from a power-law like probability density distribution

$$P(J) = D^{-1} J^{\frac{1}{D}-1}, \tag{2}$$

where $0 < J < 1$. Here, $D \geq 0$ parameterizes the disorder strength, with $D = 0$ recovering the translationally invariant case. The probability distribution (2) is a natural choice as it allows us to assess a wide range of disorder strength by varying the parameter D. Moreover, this probability distribution also coincides with the one of the infinite-randomness fixed point, which governs the critical behavior of the system [15]. Nonetheless, for the sake of completeness, we have also considered the case of box-like distributions, i.e.,

$$P(J) = \begin{cases} 1, & \text{for } J_{\min} < J < 1 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

In this case, $J_{\min}$ parameterizes the disorder strength, with smaller $J_{\min}$ meaning stronger disorder.

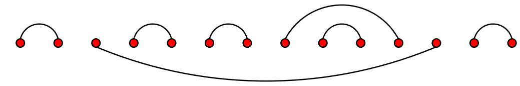

Fig. 1. Representation of the random singlet state where the red dots are the spins in a regular lattice and the black curves connect spin pairs in nearly singlet states. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

We now introduce our second model: the random correlated XX spin-1/2 chain. The difference with respect to our first model is that instead of considering an uncorrelated sequence of random couplings $\{J_1, J_2, \ldots, J_L\}$, we consider the special sequence of couplings $\{J_1, J_1, J_2, J_2, \ldots, J_{L/2}, J_{L/2}\}$. Our interest in this special model is because it was recently shown that short-range correlations among the random exchange couplings J_i (e.g., the one we are considering here, $J_{2i} = J_{2i-1}$) can dramatically change the low-energy properties of the XX spin-1/2 chain [17,18,28]. For instance, the ground state of the random correlated model is completely unrelated to the random-singlet state of the uncorrelated one; in fact, it even shares many similarities with the ground state of the translationally invariant case. For $0 \leq D \leq D_c$, the ground-state bipartite (block) entanglement and the low-energy thermodynamics are practically identical to those of the translationally invariant system [18]. Only for $D > D_c \approx 0.3$ these quantities become distinct with, surprisingly, the (block) entanglement entropy increasing with the disorder strength D (and being greater than that of the translationally invariant) [17].

3. Entanglement and Violation of Bell Inequality

In the strong-disorder limit ($D \gg 1$), it is a good approximation to describe the ground state of (1) (with uncorrelated random couplings) by the random-singlet state (see Fig. 1): a collection of independent singlets. We now would like to test this approximation by measuring how far two spins i and j are from the actual singlet state $|\Psi^-\rangle = (|+-\rangle - |-+\rangle)/\sqrt{2}$. For this reason, we study the so-called fidelity, which is given by

$$F_{ij} = \langle \Psi^- | \rho_{ij} | \Psi^- \rangle, \tag{4}$$

where ρ_{ij} is the ground-state reduce density matrix encoding all the information about the physical state of the two spins i and j. Using the symmetries of the XX spin-1/2 chain Hamiltonian, one can related the fidelity to the ground-state transverse C_{ij}^{xx} and longitudinal C_{ij}^{zz} spin-spin correlation functions [16]:

$$F_{ij} = \frac{1}{4} - 2C_{ij}^{xx} - C_{ij}^{zz}, \tag{5}$$

where $C_{ij}^{\alpha\alpha} = \langle S_i^\alpha S_j^\alpha \rangle = \mathrm{Tr}(\rho_{ij} S_i^\alpha S_j^\alpha)$. More importantly, the fidelity is related to the concurrence $\mathcal{C}_{ij}$ (a *bona fide* entanglement measurement [29–31]) via

$$\mathcal{C}_{ij} = \begin{cases} 0, & \text{if } F_{ij} \leq 1/2, \\ 2F_{ij} - 1, & \text{if } F_{ij} > 1/2. \end{cases} \tag{6}$$

Thus, for this model, the fidelity can be used as a entanglement measurement since it is monotonically related to the concurrence, with

$$F_{ij} > 1/2 \tag{7}$$

meaning that the two spins are entangled.

In addition to the entanglement, we also want to verify if the two-spins physical state is nonlocal by violating the Bell inequality $\mathcal{B}_{ij} \leq 2$ [32,33], where the Bell measurement for our model Hamiltonian is simply [3]