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Decoherence and protection of entanglement of a system of three qubits driven by a classical Gaussian distributed fluctuating field

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A B S T R A C T

We consider a system of three uncoupled entangled qubits undergoing a decoherence process (DP) induced by a classical environmental noise portrayed by a Gaussian distributed fluctuating field with either Ornstein–Uhlenbeck (OU) or Gaussian (G) autocorrelation function. The impacts of such a DP on the entanglement of the qubits are analyzed in detail when they are initialized either in the GHZ- or W-type states and interact with the fluctuating field in three different scenarios namely, common, independent and mixed environment(s). We found that: (i) the way the qubits interact with the noise as well as their initial state play an important role towards the protection of entanglement; (ii) there are optimal parameters which permit to delay or totally avoid the disentanglement of the qubits; (iii) irrespective of the qubit-noise coupling (QNC) scenario and the initial prepared state considered, the OU noise is more injurious to the survivorship of entanglement than the G one. Specifically, we show that, irrespective of the QNC scenario and the character of the noise considered, the DP disentangles the qubits more quickly when they are initialized in the W-type states than in the GHZ-type one. Furthermore, we show that when the initial state of the qubits is considered to be a W-type state, the disentanglement occurs more rapidly in the common environment (CE) scenario followed by the mixed environments (MEs) scenario than in the independent environments (IEs) one. However, the situation is completely reversed when a GHZ-type state is considered.

1. Introduction

The development of quantum technologies is based on the creation and manipulation of the quantum properties of quantum systems such as quantum correlations (entanglement or superposition of quantum states and discord). Consequently, entanglement is nowadays recognized as the most important and indispensable form of quantum correlations for performing quantum computations [1] or other task, such as, quantum teleportation [2], quantum dense coding [3] and quantum cryptography [4]. However, every quantum system always interacts with its surrounding environment and this is known as decoherence. Unfortunately, decoherence is the most obstacle to the reliable exploitation of the quantum properties of a quantum system. As a matter of fact, quantum entanglement and other correlations are very brittle and might be pulled down by the action of decoherence. Hence, exploring strategies to overcome the detrimental effects of decoherence is very important for the development of quantum technologies and this may be possible by a deep understanding of the decoherence mechanisms in quantum systems together with suitable environment engineering.

Decoherence may be simulated classically or quantum mechanically. The classical simulation, also known as stochastic model is often used to describe environments with many degrees of freedom and/or structured noise spectrum [5,7], or to describe quantum systems under the action of a classical random field [6,7]. In particular, the stochastic model of the environment is often the main effective tool available in physical, biological and social networks [8–11] to describe complex systems. On the other hand, in a more realistic situation, the environment surrounding a quantum system is constituted by a collection of many random fluctuators [5,6]. Thus, stationary and non-stationary Gaussian or non-Gaussian processes are often used to characterize the response of a quantum system under the action of an external environment. On the other hand, many experimental investigations showed that quantum systems often interact with classical

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forms of noise, especially Gaussian noise [12–14]. Although the decoherence is often described quantum mechanically, it has been shown that even certain quantum decoherence models may be in an efficient way described by their equivalent classical models [33–38]. Besides, it has been demonstrated that modeling the environment classically allows a proper description of the dynamics in the presence of memory effects, without resorting to approximated quantum master equations and that the use of classical stochastic fields enables the study of a larger class of processes standard quantum equations are not able to represent [39].

In recent years, the dynamics of quantum correlations in the situation where quantum systems are affected by environments, described classically (*i.e.*, model by means of a classical Gaussian or non-Gaussian distributed random field) has been an active area of research [5,6,15–32] and many interesting results have been obtained. In particular, it has been shown that when three qubits interact with an external classical noise such as random telegraph noise [15–17], static noise [19] and colored noise [18,20] in a CE, quantum correlations among them survive indefinitely. However, it is worth noting that the majority of these works deals with bipartite systems. This is mainly because of lack of computable quantum correlations measure for multipartite systems and also because the calculation difficulties grow up when the number of subsystem of a quantum system increases. Although the effects of Gaussian distributed classical noises on the dynamics of quantum correlations have already been investigated for bipartite systems [5], its extension to multipartite systems appears quite promising since here, quantum correlations derive from a more complex scenario, and, as a consequence, more strange phenomena are expected.

In this paper, we consider a system of three non-interacting qubits coupled with an external classical environment in three different scenarios namely: common, independent and mixed environment(s) (see figure 1 of Refs. [18,19]). In the first scenario, we suppose that the three qubits interact with the same environment; in the second scenario, we suppose that each qubit interacts with its own environment while in the third scenario two of them interact with the same environment and the remaining one with its own environment. The classical external environment is models as a Gaussian distributed stochastic field with either Ornstein–Uhlenbeck (OU) or Gaussian (G) autocorrelation function. We consider the situation where the noise spectrum of the aforementioned stochastic field contains frequencies that are larger than the typical frequencies of the qubits so that the system cannot be described as a pure dephasing. Numerical and analytical results for entanglement are presented for GHZ-type and W-type initial states by using the tripartite negativity as entanglement measure. The influences of the qubit-environment configuration as well as some parameters of the stochastic field that models the environment such as its central frequency and its correlation time on the evolution of entanglement are investigated.

The purpose of this paper is to illustrate the impacts of the DP induced by a classical environment portrayed by a Gaussian distributed stochastic field with either Ornstein–Uhlenbeck or Gaussian autocorrelation function on the evolution of entanglement of a system of three noninteracting qubit and find strategies to delay or completely suppress such a DP.

The paper is organized as follows: in section 2, we present the physical model, introduce the tripartite negativity and describe the Gaussian distributed fluctuating fields that portrayed the external environment. In section 3, we investigate the DP of the external environment. In section 4, we present the evolution of entanglement and analyze in detail the impacts of the initial prepared state of the system, the qubit-environment configuration and the nature of the fluctuating field on the evolution of entanglement. Finally, a brief discussion and summary are given in section 5.

2. Physical model

We consider a system of three noninteracting qubit interacting with a classical stochastic field (CSF) either in a CE, IEs or MEs. We suppose that the noise spectrum of the CSF contains frequencies that are larger than the natural frequency ϵ_0 of the qubit. In such a situation, the dominant process induced by the CSF is damping [7] and the single-qubit Hamiltonian may be written as:

$$\mathcal{H}_k(t) = \epsilon_{k,0}\, \sigma_k^z + g_k\Big[\vartheta_k(t)e^{-\imath\omega_k t} + \bar{\vartheta}_k(t)e^{\imath\omega_k t}\Big]\sigma_k^x, \tag{1}$$

where $\epsilon_{k,0}$ is the natural frequency of the qubit k, $\sigma_k^i, (i = x, z)$ are the Pauli matrices acting on the subspace of qubit k, g_k stands for the qubit-environment coupling constant, $\vartheta_k(t)$ is a Gaussian distributed CSF with central frequency ω_k, with a noise spectrum containing frequencies that are larger than $\epsilon_{k,0}$ and whose complex conjugate is $\bar{\vartheta}_k(t)$. It is worth nothing that $\vartheta_1(t) = \vartheta_2(t) = \vartheta_3(t)$ when the three qubits interact with a CE, meanwhile $\vartheta_1(t) \neq \vartheta_2(t) \neq \vartheta_3(t)$ when they interact with IEs. However, in the case of MEs, we suppose that $\vartheta_1(t) = \vartheta_2(t) \neq \vartheta_3(t)$. The full Hamiltonian of the three-qubit system is given by:

$$\mathcal{H}(t) = \mathcal{H}_1(t) \otimes \mathbb{I}_2 \otimes \mathbb{I}_3 + \mathbb{I}_1 \otimes \mathcal{H}_2(t) \otimes \mathbb{I}_3 + \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes \mathcal{H}_3(t), \tag{2}$$

where, $\mathbb{I}_k$ denotes the identity operator in the subspace of the k's qubit, $\mathcal{H}_k(t), (k = 1, 2, 3)$ stands for the single-qubit Hamiltonian whose explicit expression is given in Eq. (1). The model Hamiltonian in Eq. (1) represents the stochastic modeling of one of the most form of noise (phase diffusion) quantum systems are often suffer. Concretely speaking, the model under consideration may characterizes a spin-$\frac{1}{2}$ particle (whose motional degree of freedom is coupled to its quantum degree of freedom *e.g.*, its spin [7]) suffering a diffusion process in an external field having frequencies that are larger than the natural frequency of the particle.

In this model, we suppose that $\vartheta_k(t) = \vartheta_k^x(t) + \imath\vartheta_k^y(t)$ is described by a Gaussian stochastic process with zero mean and diagonal structure of the autocorrelation function. More specifically, we have

$$\begin{cases} \mathbb{E}\Big[\vartheta_k^x(t)\Big] = \mathbb{E}\Big[\vartheta_k^y(t)\Big] = 0 \\ \mathbb{E}\Big[\vartheta_k^x(t_1)\vartheta_k^x(t_2)\Big] = \mathbb{E}\Big[\vartheta_k^y(t_1)\vartheta_k^y(t_2)\Big] = \mathrm{K}(t_1, t_2) \\ \mathbb{E}\Big[\vartheta_k^x(t_1)\vartheta_k^y(t_2)\Big] = \mathbb{E}\Big[\vartheta_k^y(t_1)\vartheta_k^x(t_2)\Big] = 0 \end{cases} . \tag{3}$$

Upon assuming $t_0 = 0$, the single qubit evolution operator for a given temporal sequence of the CSF $\vartheta_k(t)$ may be expressed as: