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Moment modification, multipeakons, and nonisospectral generalizations

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Abstract

Firstly, a formal correspondence is established between the Camassa–Holm (CH) equation and a two-component modified CH (or called SQQ) equation according to the method of *moment modification* for multipeakon formulae. Secondly, based on the generalized nonisospectral CH equation in Chang et al. (2014) [14] and the interlacing multipeakons of the two-component modified CH equation in Chang et al. (2016) [15], we propose a new generalized two-component modified CH equation with two parameters, which possesses a nonisospectral Lax pair. The proposed equation still admits multipeakon solutions of explicit and closed form. Sufficient conditions for global existence of solutions are given and two concrete examples with certain interesting phenomenon are presented. Last of all, as a by-product, a generalized nonisospectral modified CH equation is deduced, together with its Lax pair.

MSC: 37K10; 35Q51; 15A15

Keywords: Moment modification; Nonisospectral Camassa–Holm type equation; Peakon solutions

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Contents

1. Introduction

The Camassa–Holm (CH) equation [12,13]

$$m_t + (mu)_x + mu_x = 0, \qquad m = u - u_{xx}, \tag{1.1}$$

is an integrable shallow water wave equation well studied in the last two decades. It firstly appeared in the work of Fuchssteiner and Fokas [28] as an abstract bi-Hamiltonian equation. But it didn't attract much more attention until it was rediscovered, by Camassa and Holm [12], and shown to admit peaked solitons (called peakons) as solutions. From then on, the CH equation has been investigated deeply and found to possess many interesting properties, such as describing breaking waves [21,35] and admitting explicitly solvable N-peakons [2,3,23], which are orbitally stable in some sense [22,38]. The other features that have attracted a lot of attention are its geometric interpretation on the Bott–Virasoro group [39,41] and its connection with the Korteweg–de Vries (KdV) equation according to tri-Hamiltonian duality [42].

The N-peakon dynamics of the CH equation is described by certain ODE system. Due to the Lax integrability, the ODE system can be solved by using inverse spectral method so that the explicit formulae of N-peakon solution to the CH equation were obtained [3]. The closed form of the solution is expressed in terms of the orthogonal polynomials with a discrete measure. Besides, it is also noted that the CH peakon flow may be projected to the finite Toda flow [4].

The discovery of CH peakons resulted in an increasing interest in search of new integrable equations. The modified Camassa–Holm (mCH) equation:

$$m_t + [m(u^2 - u_x^2)]_x = 0, \qquad m = u - u_{xx} \tag{1.2}$$