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On an advection–reaction–diffusion competition system with double free boundaries modeling invasion and competition of *Aedes Albopictus* and *Aedes Aegypti* mosquitoes

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Abstract

Based on the invasion of the *Aedes albopictus* mosquitoes and the competition between *Ae. albopictus* and *Ae. aegypti* mosquitoes in the United States, we consider an advection–reaction–diffusion competition system with two free boundaries consisting of an invasive species (*Ae. albopictus*) with density u and a local species (*Ae. aegypti*) with density v in which u invades the environment with leftward front $x = g(t)$ and rightward front $x = h(t)$. In the case that the competition between the two species is strong-weak and species v wins over species u, the solution (u, v) converges uniformly to the semi-positive equilibrium $(0, 1)$, while the two fronts satisfy that $\lim_{t\to\infty}(g(t), h(t)) = (g_\infty, h_\infty) \subset \mathbb{R}$. In the case that the competition between the two species is weak, we show that when the advection coefficients are less than fixed thresholds there are two scenarios for the long time behavior of solutions: (i) when the initial habitat $h_0 < \pi\left(\sqrt{4-\nu_1^2}\right)^{-1}$ and the initial value of u is sufficiently small, the solution (u, v) converges uniformly to the semi-positive equilibrium $(0, 1)$ with the two fronts $(g_\infty, h_\infty) \subset \mathbb{R}$; (ii) when the initial habitat $h_0 \geq \pi\left(\sqrt{4-\nu_1^2}\right)^{-1}$, the solution (u, v) converges locally uniformly to the interior equilibrium with the two fronts $(g_\infty, h_\infty) = \mathbb{R}$. In addition, we propose an upper bound and a lower bound for the asymptotic

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spreading speeds of the leftward and rightward fronts. Numerical simulations are also provided to confirm our theoretical results.

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Keywords: Invasion; Competition; Advection–reaction–diffusion model; Free boundary; Traveling wave; Asymptotic spreading speed

1. Introduction

Aedes aegypti and *Aedes albopictus* are two prominent mosquito species which transmit the viruses that cause dengue fever, yellow fever, West Nile fever, chikungunya, and Zika, along with many other diseases. *Ae. aegypti* mosquito is a species with tropical and subtropical worldwide distribution and is an insect closely associated with humans and their dwellings. *Ae. albopictus*, a mosquito native to the tropical and subtropical areas of Southeast Asia and a most invasive species, has spread recently to many countries (including the U.S.) through the transport of goods and international travel. Inter-specific competition among mosquito larvae on larval, adult, and life-table traits exists between *Ae. aegypti* and *Ae. albopictus* and affects primarily larva-to-adult survivorship and the larval development time (Noden et al. [36]).

Before the arrival of *Ae. albopictus, Ae. aegypti* was a common mosquito in artificial containers throughout Florida (Morlan and Tinker [35], Frank [17]). *Ae. albopictus* was found for the first time in northern counties in Florida in 1986 (Peacock et al. [39]). Over the next six years, *Ae. albopictus* spread slowly but steadily southward, and by the summer of 1994 it had spread to all 67 counties of the state (O'Meara et al. [37]). Meanwhile, major declines in *Ae. aegypti* abundance were associated with the invasion and expansion of *Ae. albopictus* populations, not only in Florida, but elsewhere in the southern part of the continental United States (Hobbs et al. [26], O'Meara et al. [38]). By 2008, *Ae. albopictus* had spread to 36 states and continued to expand its range (Enserink [16]). In 2013, Rochlin et al. [42] predicted that North American land favoring the environmental conditions of the *Ae. albopictus* mosquito is expected to more than triple in size in the next 20 years, especially in urban areas. By the estimates of CDC [6] in 2016, *Ae. albopictus* not only has spread to all states where *Ae. aegypti* presents but also has reached habitats beyond *Ae. aegypti*'s boundaries (see Fig. 1).

Taking into account the effect of wind on the movement of mosquitoes, Takahashi et al. [44] proposed an advection–reaction–diffusion equation model to investigate the dispersal dynamics of *A. aegypti* and predicted the existence of stable traveling waves in several situations. Although Takahashi et al. gave an estimation of the speed of traveling waves of *A. aegypti*, it is the asymptotic wave speed that usually gives an approximation of the progressive spreading speed of *A. aegypti*, and it does not really show the spread of *A. aegypti* in the early stage of spatial expanding to larger areas. To describe the spatial spreading of *A. aegypti*, we (Tian and Ruan [46]) generalized the model of Takahashi et al. [44] to an advection–reaction–diffusion equation model with free boundary, where the population of the vector mosquitoes is described by a system for the two life stages: the winged form (mature female mosquitoes) and an aquatic population (eggs, larvae and pupae), the expanding front is expressed by a free boundary which models the spatial expanding of the source area. The female mosquitoes are initially located at a habitat, then spread

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