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Gevrey regularity for the Navier–Stokes in a half-space

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Abstract

We consider the Navier–Stokes equations posed on the half space, with Dirichlet boundary conditions. We give a direct energy-based proof for the instantaneous space–time analyticity and Gevrey-class regularity of the solution, uniformly up to the boundary of the half space.

Keywords: Real analyticity; Gevrey regularity; Bounded domain; Navier–Stokes equations

1. Introduction

In this paper, we consider the Navier–Stokes system

$$\begin{aligned} &\partial_t u - \Delta u + u \cdot \nabla u + \nabla p = f \\ &\nabla \cdot u = 0, \end{aligned} \tag{1.1}$$

in the half-space $\Omega = \{x = (x_1, \dots, x_d) \in \mathbb{R}^d : x_d > 0\}$ with the no-slip boundary condition

$$u\big|_{\partial\Omega} = 0 \tag{1.2}$$

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and the initial condition

$$u(x,0) = u_0(x), \qquad x \in \Omega. \tag{1.3}$$

For simplicity, we let $d \in \{2,3\}$, but note that higher dimensions may be treated similarly. See e.g. [7,30,34] for the well-posedness and further properties of the solutions to (1.1)–(1.3).

In Theorem 1 below we prove that the solution to (1.1)–(1.3) immediately becomes space–time real analytic, with analyticity radius which is uniform up to the boundary $\partial\Omega$, under the hypothesis that the force is real analytic in space–time. The result only requires finite Sobolev regularity on the initial datum u_0.

Assuming that f is space–time analytic in $\Omega \times I$, where $\Omega \subseteq \mathbb{R}^3$ and I is a complex neighborhood of $(0,T)$, Masuda [26] proved that the interior analyticity of a solution u to the Navier–Stokes system follows from that of the external force f (see also [15]), answering a question posed by Serrin [32]. Furthermore, in the case that Ω is a bounded domain with analytic boundary $\partial\Omega$, assuming that f is analytic uniformly up to the boundary and that the solution (u,p) is C^∞, Komatsu [17,18] showed that $(u,\nabla p)$ is globally analytic in x up to the boundary $\partial\Omega$ and locally analytic in t. His technique is inspired by the previous work by Kinderlehrer and Nirenberg [16] for second order parabolic equations, and is based on an induction scheme on the number of derivatives (see also [23]). A semigroup approach for analyticity up to the boundary in (1.1)–(1.3) was later given by Giga [11] (see also [29]), and a complex variables-based proof was given by the second author and Grujić [13,14] (see also [5,6]). Establishing the analyticity of solutions to (1.1)–(1.3) on domains with boundaries is particularly important in the context of the vanishing viscosity limit [31], or equivalently, the infinite Grashof number limit in our context.

The proof of the instantaneous space–time analyticity uniformly up to the boundary of the half-space given in this paper is based solely on $L^2_{x,t}$ energy estimates of the solution and its derivatives (see also [21] for the non-homogeneous Stokes system). The main obstacle to energy-based proofs on domains with boundaries is that the normal derivatives of the solution do not obey good boundary conditions. We believe that our approach will be useful in establishing real-analytic and Gevrey-class regularization results for semilinear parabolic PDEs with different types of boundary conditions, by only appealing to energy estimates.

We recall that in the case of no boundaries, Foias and Temam have developed in [10] a very efficient method to prove analyticity, or more generally Gevrey-type regularity, which has in turn inspired many works (cf. [1–4,8,9,12,19,22,25,27,28] and references therein). The technique in [10] is based on Fourier analysis, which is unavailable in the case of domains with boundaries. One of the main aims of this paper is to find a similarly direct approach for establishing analyticity, which is based on the summing Taylor coefficients, rather than on the Fourier techniques. Such methods were introduced in [20] for the propagation of analyticity in the Euler equation, and this in turn led to be efficient in estimating the size of the uniform radius in terms of the size of initial data. However, finding an analog in the case of the Navier–Stokes equations proved to be more difficult due to the Laplacian term. In [21], the last two authors of the present paper, inspired by Komatsu's work [18], have used classical energy inequalities for the heat and Laplace equations, to achieve normal, tangential, and time derivative reductions on terms of the form $t^{i+j+k-3}\partial_t^i \partial_d^j \bar{\partial}^k u$. Here $\bar{\partial}$ and ∂_d denote the tangential derivative component and the normal derivative component, respectively. This derivative reduction method works for the heat equation and extends naturally to the inhomogeneous Stokes system, yielding the desired regularization result in [21].

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