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Infinitely many non-radial solutions to a critical equation on annulus

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Abstract

In this paper, we build infinitely many non-radial sign-changing solutions to the critical problem:

$$\begin{cases} -\Delta u = |u|^{\frac{4}{N-2}} u, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{P}$$

on the annulus $\Omega := \{x \in \mathbb{R}^N : a < |x| < b\}$, $N \geq 3$. In particular, for any integer k large enough, we build a non-radial solution which look like the unique positive solution u_0 to (P) crowned by k negative bubbles arranged on a regular polygon with radius r_0 such that $r_0^{\frac{N-2}{2}} u_0(r_0) =: \max\limits_{a \leq r \leq b} r^{\frac{N-2}{2}} u_0(r)$.

MSC: primary 35J91; secondary 35B33

Keywords: Critical exponent; Non-radial solutions; Annulus domain

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1. Introduction

This paper deals with the existence of solutions to the critical elliptic problem:

$$\begin{cases} -\Delta u = |u|^{\frac{4}{N-2}} u, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ and $N \geq 3$.

It is well known that the geometry of the domain Ω plays a crucial role in the solvability of the problem (1.1). Indeed, if Ω is a star-shaped domain, the classical Pohozaev identity [30] implies that (1.1) does not have any solutions. While if $\Omega = \{x \in \mathbb{R}^N : a < |x| < b\}$ is an annulus, Kazdan and Warner [21] found a positive solution and infinitely many radial sign-changing solutions. Without any symmetry assumptions, the existence of solutions is a delicate issue. The first existence result is due to Coron in [10] who proved that problem (1.1) has a positive solution in domain Ω with a small hole. Later, Bahri and Coron in [2] proved that actually a positive solution always exists as long as the domain has non-trivial homology with $\mathbb{Z}_2$-coefficients. However, this last condition is not necessary since solutions to problem (1.1) in contractible domains have been found by Dancer [11], Ding [17], Passaseo [28,29] and Clapp and Weth [6]. The existence of sign-changing solutions is an even more delicate issue and it is known only for domains which have some symmetries or a small hole. The first existence result is due to Marchi and Pacella [24] for symmetric domains with thin channels. Successively, Clapp and Weth [6] found sign-changing solutions in a symmetric domain with a small hole. A first attempt to remove the symmetry assumption is due to Clapp and Weth [7], who found a second solution to (1.1) in a domain with a small hole but they were not able to say if it changes sign or not. Sign-changing solutions in a domain with a small hole have been found by Clapp, Musso and Pistoia in [8]. Recently, Musso and Pistoia [25] and Ge, Musso and Pistoia [18] (see also [19]) proved that in a domain (not necessarily symmetric) with a small hole the number of sign-changing solutions to problem (1.1) becomes arbitrary large as the size of the hole decreases. The existence of a large number of sign-changing solutions in a domain with a hole of arbitrary size is due to Clapp and Pacella in [5], provided the domain has enough symmetry.

It is largely open for the problem of the existence of infinitely many sign-changing solutions in a general domain with non-trivial homology in the spirit of the famous Bahri and Coron's result.

Here, we will focus on the existence of infinitely many sign-changing solutions to problem (1.1) when $\Omega := \{x \in \mathbb{R}^N : a < |x| < b\}$ is an annulus. The existence of infinitely many radial solutions was established by Kazdan and Warner in [21]. On the other hand, an annulus is invariant under many group actions and then it is natural to expect non-radial solutions which are invariant under these group actions. Indeed, Y.Y. Li in [22] improved a previous result by Coffman [9] and he found for any integer $k \geq 1$ in a sufficiently thin annulus some non-radial solutions which are invariant under the action of the group $\mathfrak{G}_k \times \mathfrak{O}(N-2)$, when $N \geq 4$. Here $\mathfrak{O}(N-2)$ denotes the group of orthogonal $(N-2) \times (N-2)$ matrices and $\mathfrak{G}_k$ is the subgroup of matrices which rotates $\mathbb{R}^2$ with angles equal to integer multiple of $\frac{2\pi}{k}$. Recently, Clapp in [4] found infinitely many non-radial solutions which are invariant under the action of a suitable group whose orbits are infinite, provided $N = 4$ or $N \geq 6$.

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