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Biased movement and the ideal free distribution in some free boundary problems

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Abstract

Some invasive species can persist in their habitat but eventually spread very slow in a nonlinear fashion to expand their habitat range. In order to capture this phenomenon, we consider reaction–diffusion–advection models with a free boundary modeling the spreading and the biased movement of species in one-dimensional spatially heterogeneous environments. Under a condition of low resource quality, we find that large advection can lead to the spreading of the species but the spreading speed goes asymptotically to zero. Moreover, we investigate the effect of the resource on the dynamics of the current problem. Finally, we bring the notion of an ideal free distribution (IFD) into free boundary problems to understand the mechanism such that the species can eventually match the environmental quality perfectly. Under the current problem setting, the IFD may not hold even if the population plays an ideal free strategy. We then provide a sufficient condition for the IFD to be reached when using an ideal free strategy.

MSC: 35K51; 35R35; 92B05

Keywords: Free boundary problem; Biased movement; Ideal free distribution; Spreading speed

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1. Introduction

Biological invasion plays a fundamental role in understanding the dynamics of invasive species within its habitat. Since the pioneering works in the 1930s [24,33], there have been seen growing importance placed on research in investigating the wave propagation in terms of

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reaction–diffusion models. Also, the notion of asymptotic spreading speed has been particularly influential in contributing insights into theoretical invasion ecology [1,2,49,36]. Recently, a different approach to describe the spreading of an invasion species by assuming the spreading front as a free boundary has appeared that tackle the issue of biological invasion [19,8], where the Stefan condition was used to describe the propagation of the spreading front. See [40,29,37] for ecological models over bounded spatial domains involving Stefan conditions.

A classical example of invasion is the spreading of muskrats in central Europe from 1909 to 1927. The empirical data suggests that, on average, muskrat's range advanced with a constant speed [45]. In fact, the same property can be observed in various different invasion events and can be thought of as a general property of invasion. Such phenomenon has successfully been captured theoretically [44,35]. However, some species may not always spread (on average) with a constant speed. They could eventually spread very slowly to expand their habitat range even in a nonlinear fashion. For example, the spread of red deer in New Zealand from 1900 to 1950 (see [44, p. 9]) gradually slows down. It is of interest to ask if this phenomenon can be captured by mathematical models? This is our major motivation for the present study.

In the process of biological invasion, dispersal can play a key role to affect population spreading speed. Besides, dispersal is also important in determining the distributions and persistence of populations in their environment. When the environment is spatially heterogeneous, dispersal strategies have a strong impact to invasion dynamics [9,38,42]. In reality, some species can sense local environmental quality. A classical dispersal strategy proposed by Belgacem and Cosner [4] considered an advection term into logistic reaction–diffusion models with a spatially heterogeneous environment to describe that a population moves towards more favorable environment (see also [15]). Namely, the flux of the population density of species $u = u(x,t)$ at location x and time t is approximated by

$$-d\nabla u + \alpha u \nabla m,$$

where $d > 0$ is the diffusion rate and $\alpha \geq 0$ measures the rate of population movement upward along the gradient of the resource function $m(x)$. Since the movement of species may not perfectly track resource gradients in reality, more general, the flux of the population density of species can be described by

$$-d\nabla u + \alpha u \nabla P,$$

where $P(x)$ describes the movement tendency of the species, which is referred as the biased movement strategy for the species (see [12]). These considerations motivate us to consider the following problem (P):

$$u_t = [du_x - \alpha u P_x]_x + (m(x) - u)u \quad \text{for } 0 < x < s(t) \text{ and } t > 0, \tag{1.1}$$

$$du_x - \alpha u P_x = 0 \quad \text{for } x = 0 \text{ and } t > 0, \tag{1.2}$$

$$u(s(t),t) = 0 \quad \text{for } t > 0, \tag{1.3}$$

$$s'(t) = -\mu u_x(s(t),t) \quad \text{for } t > 0, \tag{1.4}$$

$$s(0) = s_0, u(x,0) = u_0(x) \quad \text{for } x \in [0,\infty), \tag{1.5}$$

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