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Ground state solutions for quasilinear Schrödinger equations via Pohožaev manifold in Orlicz space ☆

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Abstract

This paper is concerned with the following quasilinear Schrödinger equations:

$$\begin{cases} -\triangle u + V(x)u - \triangle(u^2)u = \lambda u^{p-1}u \quad \text{in } \mathbb{R}^N, \\ u > 0, \ u \in H^1(\mathbb{R}^N), \end{cases}$$

where $\lambda > 0$, $1 < p < 22^* - 1$, $2^* = \frac{2N}{N-2}$ and $N \geq 3$. Under some suitable assumptions on $V(x)$, we prove the existence of ground state solutions via Pohožaev manifold method. The novelty of this works with respect to some recent results is that we treat the existence by using Pohožaev manifold method in an Orlicz space and this enables us to handle the nonlinearity in a uniform way. As its supplementary results, we also prove the nonexistence of least energy solutions for $1 < p < 22^* - 1$.

MSC: 35J20; 35J65; 35J60

Keywords: Quasilinear Schrödinger equations; Ground state solutions; Nonexistence of least energy solutions; Orlicz space; Pohožaev manifold

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1. Introduction

This paper is to study the following quasilinear Schrödinger equations:

$$\begin{cases} -\triangle u + V(x)u - \triangle(u^2)u = \lambda u^{p-1}u & \text{in } \mathbb{R}^N, \\ u > 0, \ u \in H^1(\mathbb{R}^N), \end{cases} \tag{EQ}$$

where $\lambda > 0$ is a positive parameter, $1 < p < 22^* - 1$, $2^* = \frac{2N}{N-2}$, and $N \geq 3$. The potential $V(x)$ is continuous on $\mathbb{R}^N$, and satisfies the following hypotheses:

(v_1) $0 \leq V(x) \leq V(\infty) := \liminf_{|x|\to\infty} V(x) < +\infty$ and $V(x) \not\equiv V(\infty)$, and the inequality is strict in a subset of positive measure.
(v_2) $\langle \nabla V(x), x\rangle \in L^\infty(\mathbb{R}^N) \bigcup L^{\frac{2^*}{2^*-2}}(\mathbb{R}^N)$ and $NV(x) + \langle \nabla V(x), x\rangle \geq 0$ for all $x \in \mathbb{R}^N$.
(v_3) $V(x) \geq 0$ for all $x \in \mathbb{R}^N$, and the inequality is strict somewhere, and $\lim_{|x|\to\infty} V(x) = 0$.
(v_4) $\langle \nabla V(x), x\rangle \leq 0$ for all $x \in \mathbb{R}^N$.
(v_5) $\langle \nabla V(x), x\rangle + \frac{x\cdot H(x)\cdot x}{N} \leq 0$ for all $x \in \mathbb{R}^N$, where H represents the Hessian matrix of the function $V(x)$.

The solution of (*EQ*) is related to the existence of standing wave solutions for the quasilinear Schrödinger equation:

$$i\psi_t + \triangle\psi - W(x)\psi + k\triangle(h(|\psi|^2))h'(|\psi|^2)\psi + g(x,\psi) = 0, \forall x \in \mathbb{R}^N, \tag{1.1}$$

where $W(x)$ is a given potential, k is a real constant and $h,\ g$ are real functions. The quasilinear equation of (1.1) arises in several models of different physical phenomena corresponding to various types of h. For instance, the superfluid film equation in plasma physics has this structure for $h(s) = s$ (see [1]). In this paper, we consider standing wave solutions of (1.1) for the case $h(s) = s$ and $k = 1$.

In recent years, extensive studies have focused on the existence of solutions for quasilinear Schrödinger equations of the form:

$$-\triangle u + V(x)u - k\triangle(u^2)u = g(x,u) \quad \text{in } \mathbb{R}^N, \tag{1.2}$$

where $k > 0$ is a constant. One of the main difficulties of (1.2) is that there is no suitable space on which the energy functional is well defined and belongs to C^1-class except for $N = 1$ (see [2]). The existence of a nonnegative solution for Eq. (1.2) was proved for $N = 1$ and $g(x,u) = u^{p-1}u$ by Poppenberg, Schmitt and Wang [2] and for $N \geq 2$ by Liu and Wang in [3]. In [4], for $k = \frac{1}{2}$ and pure power nonlinearities, Liu and Wang proved that Eq. (1.2) has a ground state solution by using a change of variables and treating the new problem in an Orlicz space when $3 \leq p < 22^* - 1$ and the potential $V(x) \in C(\mathbb{R}^N, \mathbb{R})$ satisfies

(v_6) $\lim_{|x|\to\infty} V(x) = +\infty$, or ($v_7$) $V(x) = V(|x|)$ and $N \geq 2$, or
(v_8) $V(x)$ is periodic in each variable of $x_1, x_2 \ldots, x_N$, or $\lim_{|x|\to\infty} V(x) = \|V\|_{L^\infty(\mathbb{R}^N)} < \infty$.

In [5], for $k = \frac{1}{2}$ and $g(x,u) = u^{p-1}u$, Liu and Wang established the existence of both