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Traveling waves for time-delayed reaction diffusion equations with degenerate diffusion

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Abstract

This paper is concerned with time-delayed reaction–diffusion equations with degenerate diffusion. When the term for birth rate is a nonlocal integral with a heat kernel, the family of minimum wave speeds corresponding to all the degenerate diffusion coefficients is proved to admit a uniform positive infimum. However, when the term for birth rate is local, there is no positive infimum of all the minimum wave speeds. This difference indicates that the nonlocal effect plays a role as Laplacian such that a positive lower bound independent of the degenerate diffusion exists for the minimum wave speeds. The approach adopted for the proof is the monotone technique with the viscosity vanishing method. The degeneracy of diffusion for the equation causes us essential difficulty in the proof. A number of numerical simulations are also carried out at the end of the paper, which further numerically confirm our theoretical results.

Keywords: Traveling waves; Time-delay; Degenerate diffusion; Reaction–diffusion equations

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1. Introduction and preliminaries

In 1986, Metz and Diekmann [39] proposed the dynamical model of population with age-structure and diffusion:

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial a} = \widetilde{D}(a)\frac{\partial^2 v}{\partial^2 x} - \widetilde{d}(a)v, \qquad 0 < a < \tau, \tag{1.1}$$

where $v(t, x, a)$ is the population density of age a at location $x \in \Omega$ and time $t > 0$, τ is the mature time for the species, $\widetilde{D}(a)$ and $\widetilde{d}(a)$ are the diffusion rate and death rate of the population at age a. Let $u(t, x)$ be the population density of the mature at time t and point x

$$u(t, x) = \int_{\tau}^{\infty} v(t, x, a)da.$$

When the death rate d_m and diffusion rate D_m of mature population are constants, So et al. [47] derived the following reaction–diffusion equation (1.2) with nonlocal birth rate term from (1.1):

$$\frac{\partial u}{\partial t} = D_m\Delta u - d_m u + \int_{-\infty}^{+\infty} b(u(t-r, y)) f_\alpha(x-y)dy, \tag{1.2}$$

where $\alpha := \int_0^\tau D_{im}(a)da > 0$ represents the effect of the dispersal rate of immature population on the matured population and D_{im} is the diffusion rate of the immature population. f_α is the heat kernel in the form of

$$f_\alpha(y) = \frac{1}{\sqrt{4\pi\alpha}} e^{-y^2/4\alpha}, \qquad \int_{-\infty}^{\infty} f_\alpha(y)dy = 1. \tag{1.3}$$

Ecologically, since the diffusion phenomenon for the mature population at different time and different location are totally different, namely, D_m is variable and may be dependent on the population $u(t, x)$, so it is more practical and reasonable for us to consider the following time-delayed nonlinear diffusion equation:

$$\frac{\partial u}{\partial t} = \nabla(\varphi(u)\nabla u) - d(u) + \int_{-\infty}^{+\infty} b(u(t-r, y)) f_\alpha(x-y)dy. \tag{1.4}$$

Here, the diffusion of mature species is considered to be degenerate in the form of $-\nabla(\varphi(u)\nabla u)$ with $\varphi(u) = Dmu^{m-1}$ and $m > 1$, which is dependent on the population density due to the population pressure. See also the derivation and background stated later. Such a degenerate diffusion means that the smaller density, the slower spatial-diffusion, particularly, zero density implies non-diffusion. D represents the diffusivity of the mature population, and $d(u)$ is the death rate function in a general form.

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