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L^2-norm blow-up of solutions to a fourth order parabolic PDE involving the Hessian ☆

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Abstract

This paper deals with a fourth order parabolic PDE arising in the theory of epitaxial growth of crystal. We focalize the study on one open question proposed by Escudero et al. (2015) [4], that is, L^p-norm blow-up. For the initial energy $J(u_0) < \frac{\lambda_1}{6}\|u_0\|_2^2$, we prove the solution blows up in finite time with L^2-norm, where λ_1 is the least Dirichlet eigenvalue of the biharmonic operator. Moreover, the lifespan of the solution is got, and the results of this paper also generalize the results got by Xu and Zhou (2017) [12].

MSC: 35J30; 35K25; 35K35; 35K55; 35G31; 35Q70

Keywords: Epitaxial growth; Forth order parabolic PDE; Blow-up; Lifespan

1. Introduction

Let $\Omega \subset \mathbb{R}^2$ be an open, bounded domain with smooth boundary $\partial\Omega$ and denote the element of $\mathbb{R}^2$ by (x, y). In this paper, we consider the following fourth order parabolic PDE:

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$$\begin{cases} \dfrac{\partial u}{\partial t} + \Delta^2 u = \det\left(D^2 u\right), & (x,y) \in \Omega, t > 0, \\ u((x,y),0) = u_0(x,y), & (x,y) \in \Omega, \\ u((x,y),t) = \dfrac{\partial u}{\partial \nu}((x,y),t) = 0, & (x,y) \in \partial\Omega, t > 0, \end{cases} \tag{1.1}$$

where

$$\begin{cases} \Delta^2 u := \Delta(\Delta u) = u_{xxxx} + u_{yyxx} + u_{xxyy} + u_{yyyy}, \\ \det\left(D^2 u\right) := \begin{vmatrix} u_{xx} & u_{xy} \\ u_{yx} & u_{yy} \end{vmatrix} = u_{xx}u_{yy} - u_{xy}u_{yx}, \end{cases}$$

the initial value $u_0(x,y) \in W_0^{2,2}(\Omega)$ and ν is the unit out normal vector on $\partial\Omega$.

Problem (1.1) describes the process of epitaxial growth [2,11], which was studied in [1,3–7,9,12]. Especially, in [4], the authors studied the behavior of the solutions, and they proposed seven open questions, the main purpose of this paper is to study one of them.

In order to introduce the question in detail, let's firstly introduce some notations, sets and functionals given in [4].

For $p \in [1, +\infty)$, we denote by $\|\cdot\|_p$ the L^p-norm and

$$\|u\|_p = \left(\int_\Omega |u|^p dxdy\right)^{\frac{1}{p}}.$$

The norm of $W^{m,p}(\Omega)$ is denoted by $\|\cdot\|_{W^{m,p}(\Omega)}$ except for the $W_0^{2,2}$-norm, which is denoted by $\|\cdot\|$, and

$$\|u\| = \|\Delta u\|_2.$$

Let λ_1 be the least eigenvalue of the following eigenvalue problem:

$$\begin{cases} \Delta^2 u = \lambda u, & (x,y) \in \Omega, \\ u(x,y) = u_\nu(x,y) = 0, & (x,y) \in \partial\Omega. \end{cases} \tag{1.2}$$

By [8], we know that λ_1 is positive, simple and it can be characterized in the following variational form:

$$\lambda_1 = \inf_{u \in W_0^{2,2}(\Omega)\setminus\{0\}} \frac{\|u\|^2}{\|u\|_2^2}, \tag{1.3}$$

which implies

$$\lambda_1 \|u\|_2^2 \le \|u\|^2, \quad \forall u \in W_0^{2,2}(\Omega). \tag{1.4}$$

Next, we define the energy functional related to the stationary equation of (1.1) by

$$J(u) := \frac{1}{2}\|u\|^2 - I(u), \quad \forall u \in W_0^{2,2}(\Omega), \tag{1.5}$$