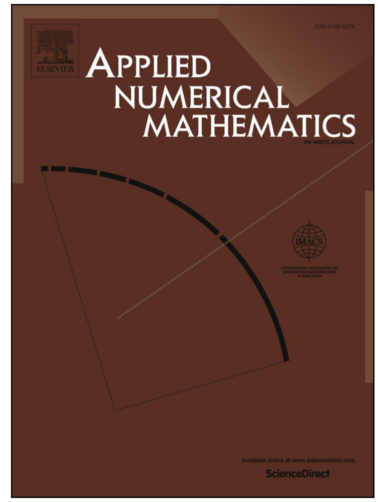

A stabilized finite volume element method for a coupled Stokes-Darcy problem

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A stabilized finite volume element method for a coupled Stokes-Darcy problem *

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Abstract

In this paper, we present a stabilized finite volume element method with the conforming finite element triples $P_1 - P_0 - P_1$ and $P_1 - P_1 - P_1$ for approximating the velocity, pressure, and hydraulic head of a coupled Stokes-Darcy problem. The proposed method is convenient to implement, computationally efficient, mass conserving, optimally accurate, and able to handle complex geometries; therefore, this method has great potential to be useful for realistic problems involving coupled free flow and porous media flow. To offset the lack of the inf-sup condition of the $P_1 - P_0$ and $P_1 - P_1$ elements for the Stokes equation, a parameter free stabilization term is added to the discrete formulation. Stability and optimal error estimates are proved based on a bridge built up between the finite volume element method and the finite element method. An element level implementation of the stabilization term is discussed so that an existing code package can be conveniently modified to handle the stabilization procedures. A series of numerical experiments are provided to illustrate the above features of the proposed method, the theoretical results, and the realistic applications.

Keywords: Coupled Stokes-Darcy flow; finite volume element method; stability; Beavers-Joseph-Saffman-Jones condition

1 Introduction

The conforming finite volume element method (FVEM) is a highly effective numerical method for partial differential equations, and therefore it has been extensively studied and widely applied to different types of problems, see [5, 10, 11, 19–21, 24, 25, 27, 49, 57, 63, 75, 76] and references therein. The method combines the strengths of the finite volume and finite element methods. Specifically, as in the finite volume method, the FVEM is based on local conservation of mass, momentum, or energy. Also, as in the finite element method, the FVEM can easily deal with complicated geometries while also obtaining the optimal accuracy expected from the polynomials utilized for the finite element basis functions.

We consider the Stokes-Darcy model for coupling fluid flow in conduits with porous media flow. This type of coupled flow is often involved in many applications, such as subsurface flow problems [17, 18, 29, 47, 54], industrial filtrations [44], and flow in vuggy porous media [2]. The model consists of Stokes equations to govern the flow in conduits, Darcy's law to govern the flow in porous media, and three interface conditions to couple these two constituent models together.

Due to the complexity of this model, many methods have been developed to numerically solve the Stokes-Darcy system, such as coupled finite element methods [12, 16, 48, 52, 59, 62], domain decomposition

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