ARTICLE IN PRESS
APNUM:3311

Applied Numerical Mathematics ••• (••••) •••–•••

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

An adaptive nonlinear elimination preconditioned inexact Newton algorithm for highly local nonlinear multicomponent PDE systems

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Fluid flow
Heat transfer
Adaptive nonlinear elimination
Highly local nonlinearity
Inexact Newton method

ABSTRACT

This work aims to develop an adaptive nonlinear elimination preconditioned inexact Newton method as the numerical solution of large sparse multi-component partial differential equation systems with highly local nonlinearity. A nonlinear elimination algorithm used as a nonlinear preconditioner has been shown to be a practical technique for enhancing the robustness and improving the efficiency of an inexact Newton method for some challenging problems, such as the transonic full potential problems. The basic idea of our method is to remove some components causing troubles in order to decrease the impact of local nonlinearity on the global system. The two key elements of the method are the valid identification of the to-be-eliminated components and the choice of subspace correction systems, respectively. In the method, we employ the point-wise residual component of nonlinear systems as an indicator for selecting these to-be-eliminated components adaptively and build a subspace nonlinear system consisting of the components corresponding to the bad region and an auxiliary linearized subsystem to reduce the interfacial jump pollution. The numerical results demonstrate that the new approach significantly improves performance for incompressible fluid flow and heat transfer problems with highly local nonlinearity when compared to the classical inexact Newton method.

1. Introduction

A class of nonlinear preconditioned iterative algorithms, namely the additive Schwarz preconditioned inexact Newton algorithm (ASPIN) proposed by Cai and Keyes [4], opened up a new research direction on the development of nonlinear iterative solvers. In the past, the typical application of a preconditioner for nonlinear problems was usually employed together with a Krylov subspace method for the numerical solution of the Jacobian systems, e.g., the Newton–Krylov–Schwarz algorithm (NKS) [3,21] and the Newton–Krylov-multigrid algorithm [22,27], or for other similar linearized systems arisen from the Picard-type iterative method. The major role of linear preconditioners is to accelerate the convergence of an iterative method to obtain a high-quality Newton search direction, but it usually has nothing to do with helping to improve the robustness of an inexact Newton method if no good initial guess is available. On the other hand, ASPIN constructs the

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https://doi.org/10.1016/j.apnum.2018.01.008
0168-9274/

Please cite this article in press as: H. Yang, F.-N. Hwang, An adaptive nonlinear elimination preconditioned inexact Newton algorithm for highly local nonlinear multicomponent PDE systems, Appl. Numer. Math. (2018), https://doi.org/10.1016/j.apnum.2018.01.008

nonlinear preconditioner based on the overlapping Schwarz framework and applies it directly to the nonlinear system. In this case, the convergence of an inexact Newton method is not sensitive to the selection of an initial guess, physical parameters, e.g., the Reynolds number for the incompressible flows and the Mach number for the compressible flows, and the system parameters, e.g., geometric configuration, mesh sizes, etc. The basic idea of ASPIN is to reformulate the original nonlinear system as some easier-to-solve preconditioned nonlinear system implicitly and then solve the new system through an inexact Newton method. After more than ten years of progress, Cai and Keyes' work now draws much attention from the scientific community. Among them, ASPIN and its variants such as the multiplicative Schwarz version [24–26] or the restricted additive Schwarz version [12] have been applied successfully to incompressible high-Reynolds number flows [4,5,7,17,18], high-Rayleigh number convection flows [26], transonic compressible flows [6,19,38], multiphase flows in porous media [30,32], unconstrained optimization problems arising in nonlinear elasticity problems [16], and image processing [39]. However, the nonlinear additive Schwarz preconditioner in the ASPIN belongs to the class of the *left* nonlinear preconditioners, which has some drawbacks. For example, the computational cost for the numerical evaluation of the nonlinear preconditioned function can be very expensive, since it requires to solve several subdomain nonlinear problems, although it can easily be parallelized due to the independence of these subdomain problems. The overhead of ASPIN can be more significant compared to the NKS algorithm, especially when the certain global Newton iteration requires to perform few backtracking steps, a commonly-used globalization technique to assure sufficient progress of Newton iteration. Hence, ASPIN is intended for the cases where classical Newton-type method fails to converge.

On the other hand, nonlinear preconditioning can be applied to the right of the nonlinear function. Instead of changing the function of the system itself as a left nonlinear preconditioner does, right preconditioning modifies the unknown variables of the nonlinear system. Also, right nonlinear preconditioning can be interpreted as nonlinear coordinate transformation between the solution spaces, see Ref. [35], where Yang et al. used a simple example to illustrate this idea. Hwang et al. [19,20] employed a nonlinear elimination (NE) technique [23] as a right preconditioner for the 1D and 2D transonic full potential flow problems. The basic idea of NE is to remove these components that cause trouble for IN implicitly. An effective identification of the to-be-eliminated variable set is the critical step for the success of the overall algorithm. One possible strategy to determine which components to eliminate based on a priori knowledge or feedback from the intermediate numerical solution. For example, the numerical local Mach number provides such useful information, where the shock wave is located, which corresponds to the highly local nonlinear components to be eliminated in the transonic flow calculations [20].

The objectives of this work are to develop a class of adaptive NE preconditioners for the multicomponent systems with highly local nonlinearities and to study numerically the robustness and efficiency of the IN method preconditioned by NE for some challenging flow problems, including high-Reynolds number forced and mixed convection cavity flow problems. In general, designing effective elimination strategies for multicomponent systems is not trivial, since each variable interacts with the others. One possible elimination strategy is based on the field variable. For example, Lanzkron et al. [23] considered the drift-diffusion equations consisting of the Poisson equation and both the electron and hole continuity equations for the semiconductor device simulation. Their numerical result showed that when the field variables corresponding to the two continuity equations are selected to eliminate, the efficiency of the classical IN is significantly improved by using together with the nonlinear elimination technique. Further successful application examples of the field variable based NEs were the thermal convective flow control problem [35] and the two-phase porous media flow problem [37]. On the other hand, we adopt the domain decomposition elimination approach, i.e., when one variable on some particular mesh point is selected to eliminate, all other variables corresponding to that mesh point are also eliminated. In this work, we perform some numerical experiments to obtain some insight about how to design the elimination process and study several different elimination strategies.

The remainder of the paper is structured as follows. In Section 2, we give a general description of the *right* nonlinear elimination preconditioned inexact Newton algorithm for PDE problems with highly local nonlinearity. In Section 3, we introduce the governing equations for fluid flows and heat transfer, which are our target applications, together with their discretizations. In Section 4, we introduce the nonlinear elimination preconditioner for the multicomponent PDEs. In Section 5, we present some numerical results for two benchmark problems, including lid-driven cavity flow problem and forced convective heat transfer problem. We conclude this paper in Section 6.

2. A framework of right NE preconditioned inexact Newton algorithm

We consider an inexact Newton method with backtracking technique (INB) [11,13] in conjunction with a right nonlinear elimination preconditioner for finding a root of the large, sparse, nonlinear system of equations,

$$F(x) = 0, \tag{1}$$

where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a given nonlinear vector-valued function arising from some discretization of PDE given by $F = (F_1, F_2, \cdots, F_n)^T$ with $F_i = F_i(x_1, x_2, \cdots, x_n)^T$, and $x = (x_1, x_2, \cdots, x_n)^T$. To begin with, let us introduce a right nonlinear preconditioned system [35]

$$W(y) \equiv F(G(y)) = 0, \tag{2}$$