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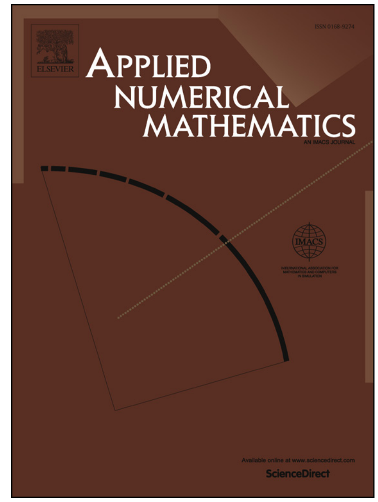

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OPTIMAL ERROR ESTIMATES OF BOTH COUPLED AND TWO-GRID DECOUPLED METHODS FOR A MIXED STOKES-STOKES MODEL*

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Abstract. In this paper, we provide a coupled algorithm and a two-grid decoupled algorithm for a mixed Stokes-Stokes model, which is coupled by a nonlinear interface transmission condition. The coupled algorithm is to discretize the mixed model directly by standard finite element method. For the two-grid decoupled algorithm, we first solve the mixed model on a coarse grid, and update the solution on a fine grid by two separated Stokes problems. Under a hypothesis about the regularity of analytical solutions, optimal error estimates for two algorithms are achieved. Several numerical tests are given to verify our theoretical results.

Key words. coupled Stokes problem, decoupled method, two-grid method, nonlinear interaction condition, optimal error estimates.

AMS subject classifications. 65N30, 65M60, 76D07, 76M10.

1. Introduction. Numerical methods for two turbulent flows in different regions coupled by a nonlinear interface transmission condition were studied in [2, 3, 4, 10]. They are widely used in engineering to predict the behavior of many kinds of flows with applied interest.

The model of mixed Stokes-Stokes flows is a simplified form of two turbulent flows by removing the turbulent kinetic energy. In [9], the author considered the stationary mixed Stokes-Stokes flows. Since it is a time consuming procedure to calculate the coupled systems directly by standard finite element method, they proposed the Uzawa type domain decomposition algorithm to reduce the high computational cost. Concretely speaking, by introducing a fictitious variable in the interface transmission condition and using the saddle point theory, the author derived two Uzawa-type domain decomposition algorithms. In that way, only two uncoupled Stokes problems or two uncoupled Poisson problems need to be solved on each iteration step.

For the non-stationary case, a coupled Navier-Stokes fluids with nonlinear interface transmission condition was studied in [5, 10, 22]. For example, by adopting an operator-splitting and optimization-based non-overlapping domain decomposition methods, the authors in [5] solved one coupled degenerated Stokes problem in the first sub-step and one uncoupled linear advection-diffusion problem in the second sub-step. Moreover, the authors in [10] proposed two decoupled algorithms by implicit-explicit approach. They only analyzed the stability and error estimates for their second algorithm and left the stability of their first algorithm as an open question. Recently, the authors in [22] derived the stability and convergence results for the first decoupled algorithm in [10]. Besides, in [23], the author analyzed stability of a decoupled scheme for a coupled Navier-Stokes fluids with linear interface condition and obtained its convergent rate.

There are several attractive reasons that have led to active research on developing effective and efficient decoupling techniques for multi-physics mixed models so that existing single-model solvers can be applied locally with little extra computational and software overhead. For the mixed model with different types of the fluid in each subdomain, many efficient decoupled methods have

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