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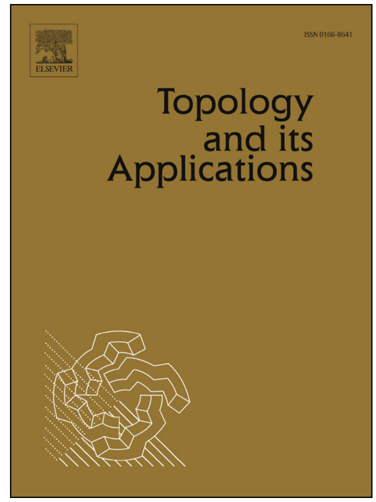

Relative topological complexity of a pair

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RELATIVE TOPOLOGICAL COMPLEXITY OF A PAIR

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Abstract. For a pair of spaces X and Y such that $Y \subseteq X$, we define the relative topological complexity of the pair (X, Y) as a new variant of relative topological complexity. Intuitively, this corresponds to counting the smallest number of motion planning rules needed for a continuous motion planner from X to Y. We give basic estimates on the invariant, and we connect it to both Lusternik-Schnirelmann category and topological complexity. In the process, we compute this invariant for several example spaces including wedges of spheres, topological groups, and spatial polygon spaces. In addition, we connect the invariant to the existence of certain types of axial maps.

1. Introduction

Topological complexity (TC) is an invariant introduced by Michael Farber in [Far03] connecting a motion planning problem in robotics with algebraic topology. Intuitively, we think of topological complexity as the smallest number of "rules" needed to form a continuous motion planning algorithm on a topological space X. For robotics, we think of X as the configuration space of some robot, and our motion planning algorithm outputs paths between the configurations in X. Continuity then requires that the paths remain "close" when the configurations are "close" in some sense. It turns out that $\mathrm{TC}(X)$ only depends on the topology of X, and so topologists study the invariant applied to various topological spaces in the abstract sense rather than as configuration spaces of specific robots.

In the years since its introduction, several different variations of topological complexity have been studied pertaining to different motion planning problems. Of interest to us is relative topological complexity. Here, we restrict which configurations are allowed to be starting and ending configurations, but we permit the path to move within a larger configuration space. This variant is mentioned in Farber's book on the subject [Far08], and it is used there to prove that $\mathrm{TC}(X)$ is a homotopy invariant. In this paper, we restrict our attention further to a certain method for choosing starting and ending configurations.

Our invariant is motivated by the following motion planning problem. Suppose there was a robot with configuration space X, and the robot is given to us in an arbitrary configuration within X. Our goal is to plan the robot's motion to a configuration within some specified subset $Y \subseteq X$. Then the relative topological complexity of the pair (X, Y) is the smallest number of "rules" needed to form a continuous motion planning algorithm on X where the paths must end in Y. This restriction provides two major advantages. First, we are able to develop some standard tools for estimating this value, which we do in Section 2. Then, there are natural relationships between this value and both $\mathrm{TC}(X)$ and the Lusternik-Schnirelmann category of X which we explore in Section 3. Throughout this paper, we will assume our spaces are path-connected unless otherwise specified.

In Section 4, we apply this new variant of relative topological complexity to pairs of real projective spaces. In so doing, we draw a deep connection to the existence of certain axial maps. We draw this connection explicitly in Theorem 4.3. This connection follows a similar logic to [FTY03], where Farber, Tabachnikov, and Yuzvinsky connect the immersion dimension of real projective spaces to their topological complexity using axial maps.

Finally, in Section 5, we apply this new variant to pairs of spatial polygon spaces. These have been studied by Hausmann and Knutson in [HK98] as well as by Davis in [Dav16]. In our study, we introduce new notation for interesting submanifolds of the spatial polygon spaces for consideration

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