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An accurate numerical solution for the singular velocity field near the maximum friction surface in plane strain extrusion

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ABSTRACT

In the case of rigid perfectly plastic material, the velocity field is singular in the vicinity of maximum friction surfaces (the equivalent strain rate approaches infinity in the vicinity of such surfaces). This causes significant difficulties with the convergence of finite element solutions. On the other hand, an accurate description of the velocity field near frictional interfaces is crucial for predicting the generation of a narrow fine grain layer that frequently appears near such interfaces. The present paper provides an accurate numerical solution for the singular velocity field in the vicinity of the maximum friction surface in plane strain extrusion. The singularity in the velocity field is represented by means of the strain rate intensity factor. The numerical approach is based on the method of characteristics. The strain rate intensity factor is determined by means of simple formulae once the solution for the radii of curvature of characteristic lines and velocities has been found. It is shown that the distribution of the strain rate intensity factor along the friction surface is discontinuous. Comparison with an analytic solution for material flow through an infinite wedge-shaped die is made and a high accuracy of the numerical solution is demonstrated. On the other hand, it is shown that the analytic solution is not accurate enough to find the strain rate intensity factor even for rather long dies.

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1. Introduction

Narrow fine grain layers of material often appear in the vicinity of frictional interfaces in manufacturing processes as a result of severe shear plastic deformation (Griffiths, 1987; Kajino and Asakawa, 2006). Mechanical material behavior within such layers is far removed from that encountered in conventional material tests and, therefore, conventional constitutive equations are not applicable. A similar problem occurs in finding mechanical behavior of material in metal cutting (Jaspers and Dautzenberg, 2002). A novel approach to predicting the generation of fine grain layers near frictional interfaces in metal forming processes has been proposed in Alexandrov and Goldstein (2015) and Goldstein and Alexandrov (2015). This approach is based on the strain rate intensity factor introduced in Alexandrov and Richmond (2001). The strain rate intensity factor controls the magnitude of the equivalent strain rate in the vicinity of maximum friction surfaces. In the metal forming literature, it is often assumed that this friction law represents sticking friction conditions (Wang, 2001; Chandrasekharan et al., 2005). However, it is worthy of note that the actual seizure (the velocity vector is continuous across the interface) may or may not occur in theoretical solutions. The approach proposed in Alexandrov and Goldstein (2015) and Goldstein and Alexandrov (2015) requires nonconventional experimental and numerical methods. Experimental research in this direction has been initiated in Alexandrov et al. (2015) and Hwang et al. (2015). These preliminary experimental results have demonstrated the potential of the approach.

A difficulty with numerical solutions is that the strain rate intensity factor is the coefficient of the leading singular term in a series expansion of the equivalent strain rate in the vicinity of maximum friction surfaces. In particular, the equivalent strain rate approaches infinity near such surfaces for several rigid plastic material models (Alexandrov and Richmond, 2001; Alexandrov and Lyamina, 2002; Alexandrov and Jeng, 2013; Alexandrov and Mustafa, 2015). For this reason commercial finite element packages are not capable of calculating the strain rate intensity factor

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(Facchinetti and Miszuris, 2016). In particular, finite element solutions do not converge when the maximum friction law is adopted (Rebelo and Kobayashi, 1980; Chen et al., 1998). In general, conventional numerical methods fail to predict high velocity gradients that in fact appear in the vicinity of interfaces between tool and workpiece with high friction (Appleby et al., 1984). Probably, the extended finite element method (Fries and Belytschko, 2010) can be used for calculating the strain rate intensity factor. However, to the best of authors' knowledge, no attempt has been made to apply this method.

A new efficient numerical procedure based on the method of characteristics has been recently proposed for calculating the strain rate intensity factor in planar flow of rigid perfectly plastic material in Alexandrov et al. (2016). Using this procedure the strain rate intensity factor has been found in compression of a plastic layer between two parallel plates in Alexandrov et al. (2016). The accuracy of the numerical solution has been evaluated by comparison with the strain rate intensity factor found by means of an analytic solution given in Hill (1950). In the present paper, the numerical procedure proposed in Alexandrov et al. (2016) is adopted to find an accurate solution near the friction surface and to determine the strain rate intensity factor in plane strain extrusion through a wedge-shaped die. This process is ideal for experimental research that is required to develop the approaches proposed in Alexandrov and Goldstein (2015) and Goldstein and Alexandrov (2015) since severe shear plastic deformation in the vicinity of the friction surface is easily generated in extrusion.

Many numerical schemes based on the method of characteristics have been already developed for solving plane strain boundary value problems for rigid perfectly plastic solids (Hill et al., 1951; Ewing, 1967, 1968; Collins, 1968; Bachrach and Samanta, 1976). The procedure for calculating the strain rate intensity factor proposed in Alexandrov et al. (2016) can be used in conjunction with any numerical scheme based on the method of characteristics. In the present paper, the scheme developed in Hill et al. (1951) is adopted. Then, the strain rate intensity factor is found for plane strain extrusion. The validity and accuracy of the procedure for calculating the strain rate intensity factor is verified. Other slip-line solutions for the process of plane strain drawing/extrusion have been given in Hill (1950), Bishop (1953), Hillier (1962), Rogers and Coffin (1971), Dodd and Kudo (1980) and Chakrabarty (1987). The strain rate intensity factor has not been calculated in these works.

2. The strain rate intensity factor in characteristic coordinates

In the case of arbitrary three-dimensional flow of rigid perfectly plastic material the equivalent strain rate in the vicinity of maximum friction surfaces is represented as (Alexandrov and Richmond, 2001)

$$\xi_{eq} = \sqrt{\frac{2}{3}\xi_{ij}\xi_{ij}} = \frac{D}{\sqrt{s}} + o\left(\frac{1}{\sqrt{s}}\right) \tag{1}$$

as $s \to 0$. Here ξ_{ij} are the physical components of the strain rate tensor, ξ_{eq} is the equivalent strain rate, D is the strain rate intensity factor, s is the normal distance to the maximum friction surface, and o is the order symbol. The definition for maximum friction surfaces is that the friction stress at sliding is equal to the shear yield stress, k. The latter is a material constant in the case of rigid perfectly plastic material. Eq. (1) is not valid for some special flow. In particular, the plane strain boundary value problem is described by a hyperbolic system of equations (Hill, 1950). In this case Eq. (1) is valid if and only if the friction surface coincides with an envelope of characteristics. In what follows, it is assumed that this condition is satisfied.

A great account on the classical theory of rigid perfectly plastic solids is given in Hill (1950). The expression for the strain rate intensity factor in characteristic coordinates has been derived in Alexandrov et al. (2016). For completeness, the basic equations that are necessary for calculating the strain rate intensity factor numerically are summarized below. In the case of plane strain deformation, the characteristic lines can be regarded as a right handed curvilinear orthogonal coordinate system $(\alpha,\ \beta)$. By the convention, the orientation of these lines is chosen such that the algebraically greatest principal stress, σ_1, falls in the first and third quadrants. The other principal stress in $\alpha\beta-$planes will be denoted by σ_2. In general, one or two families of characteristics can be straight. These special cases are excluded from consideration. Then, the system of equations to solve is

$$\frac{\partial S}{\partial \alpha} + R = 0, \quad \frac{\partial R}{\partial \beta} - S = 0 \tag{2}$$

and

$$\frac{\partial u_\alpha}{\partial \alpha} - u_\beta = 0, \quad \frac{\partial u_\beta}{\partial \beta} + u_\alpha = 0. \tag{3}$$

Here R is the radius of curvature of the $\alpha-$lines, S is the radius of curvature of the $\beta-$lines, u_α and u_β are the velocity components referred to the $\alpha-$and $\beta-$lines, respectively. Let ϕ be the anti-clockwise angular rotation of the $\alpha-$line from the $x-$axis of a Cartesian coordinate system (x, y). Then,

$$\phi = \alpha + \beta \tag{4}$$

Since the maximum friction law dictates that the friction surface coincides with an envelope of characteristics, this boundary condition can be represented as

$$R = 0 \text{ or } S = 0 \tag{5}$$

on the friction surface. The asymptotic representations of R and S in the vicinity of maximum friction surfaces are (Alexandrov et al., 2016)

$$R = \pm\frac{R_0^2}{2}(\alpha - \alpha_Q) + o(\alpha - \alpha_Q) \tag{6}$$

as $\alpha \to \alpha_Q$ and

$$S = \pm\frac{S_0^2}{2}(\beta - \beta_Q) + o(\beta - \beta_Q) \tag{7}$$

as $\beta \to \beta_Q$. Here α_Q is the value of α at a generic point of the maximum friction surface on which $R=0$ and β_Q is the value of β at a generic point of the maximum friction surface on which $S=0$. The strain rate intensity factor is given by (Alexandrov et al., 2016)

$$D = \frac{1}{\sqrt{3}|S_0|}\left|-\frac{\partial u_\alpha}{\partial \beta} + u_\beta\right| \tag{8}$$

at the maximum friction surface on which $S=0$ and

$$D = \frac{1}{\sqrt{3}|R_0|}\left|\frac{\partial u_\beta}{\partial \alpha} + u_\alpha\right| \tag{9}$$

at the maximum friction surface on which $R=0$. In Eqs. (8) and (9), the quantities $\partial u_\alpha/\partial\beta$, $\partial u_\beta/\partial\alpha$, u_β, and u_α are understood to be calculated at points of the friction surface. Thus, once a solution to Eqs. (2) and (3) has been found in the characteristic coordinates, the strain rate intensity factor is immediately determined from (8) or (9). In particular, it is seen from (6) and (7) that R_0 and S_0 are involved in the coefficients of the linear terms in the series expansions of R and S in the vicinity of maximum friction surfaces in the characteristic coordinates.

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