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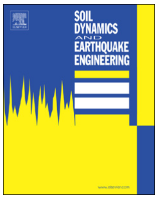

Random vibration analysis of multi-floor buildings using a distributed parameter model

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ABSTRACT

The objective of the current paper is to provide an accurate distributed parameter model for random vibration analysis of multi-floor buildings. The Hamilton's principle is employed to derive the equations governing the dynamic behavior of the system as well as the related kinematic and natural boundary conditions. The natural frequency and mode shapes of the developed model are then extracted analytically and validated using finite element simulations. It is also observed that the predictions of the proposed model for the natural frequencies of the system is far more accurate than those of that of the discrete model available in the literature. Using a single mode approximation in the Lagrange equation, the partial differential equations of the motion are reduced to a single ordinary differential equation. Assuming a band limited white noise for the acceleration of the support, the random response specifications (such as expected value, autocorrelation, spectral density and mean square) of the system is calculated by making use of the random vibration theory. The qualitative and quantitative nature of the response characteristic are also analyzed to reveal the effects of different design parameters on the system's response. The suggested modeling approach in this paper may be employed for prediction of the dynamic behavior of more complex structures to different types of deterministic or random excitations. Also the provided analytical method for the random response calculation of the system can be utilized to make more informed decisions in the design process.

1. Introduction

Protection of engineering structures against un-wanted vibrations have always been a challenge for civil and mechanical engineers. In building constructions, an earthquake can bring about severe unwelcomed vibrations of the system and produce large stresses which ultimately can lead to catastrophic collapse of the structure. So researchers have been trying to innovate new techniques to study the vibrational phenomena in such structures and also to develop new vibration suppression techniques to minimize their vibration level.

In some studies, the earthquakes have been modeled as harmonic support motion. For example Farshidianfar and Soheili [1] investigated the optimized parameters of tuned mass dampers for high rise structures considering soil structure interaction effects under harmonic base excitations. Park and Reed [2] examined the performance of uniformly and linearly distributed multiple mass dampers in suppressing the vibrations resulted from harmonic and earthquake excitations.

In practice, the nature of earthquake is not deterministic. So in many other studies, the vibrational response of buildings and other mechanical systems has been analyzed based on the hypothesis of random excitations. For example, Kiureghian and Neuenhofer [3] developed a new response spectrum method for seismic analysis of linear multi-degree-of-freedom, multiply supported structures subjected to spatially varying ground motions. Heredia-Zavoni and Vanmarcke [4] employed the random-vibration methodology to study the seismic random response analysis of linear multi support structural systems. While respecting the stationarity assumption, their method, simplified the random analysis by equalizing the response evaluation of the system to that of a series of linear one-degree systems. Wen [5] provided an overview of the major developments in modeling and response analysis of inelastic structures under random excitations. Kiureghian [6] developed a response spectrum method for stationary random vibration analysis of linear, multi-degree-of-freedom systems. His method was based on the assumption that the input excitation is a wide-band, stationary Gaussian excitation and the response. Radeva and Radev [7] proposed a method for simulation of seismic ground motions. They considered the random motion as a stationary filtered white noise with fuzzy parameters and proposed an analytical procedure to analyze the

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fuzzy random vibration of multi-degree-of-freedom hysteretic buildings. Coupling adjacent buildings with supplemental damping devices is a practical and effective approach to mitigating structural seismic response. Hao and Zhang [8] used the random vibration method to study earthquake ground motion spatial variation effects on relative linear elastic response of adjacent building structures. Ni et al. [9] developed a method for analyzing the random seismic response of a structural system consisting of two adjacent buildings interconnected by hysteretic damping devices. Effective elimination of the vibration of building structures due to earthquake and wind loading via passive, semi active and active structures have also been investigated in the prior art. Ikeda and Ibrahim [10] investigated the passive vibration control of an elastic structure carrying a rectangular tank partially filled with liquid, subjected to horizontal narrow band random ground excitation. Yang et al. [11] studied vibration suppression of structures using a semi-active mass damper under random base excitation. Gur and Mishra [12] presented the optimal stochastic performance of pure friction system supplemented with shape memory alloy assisted pure friction, based on a framework of multi-objective optimization. Ozbulut and Hurlebaus [13] proposed a new device which took advantages of both variable friction dampers as well as shape memory alloys to intelligently suppress the vibration of a building structure. Calise et al. [14] carried out a series of experiments to quantify the potential benefits of using robust control design methodology in active control of building structures.

Many researchers studied random vibration analysis of structures to non-stationary inputs. Alderucci and Muscolino [15] presented a closed-form solution for the evolutionary power spectral density of the response of linear classically damped structural systems subjected to fully non-stationary multi-correlated excitations. They utilized their method to study random vibration of a bridge, and validated their findings with Monte Carlo Simulations. Muscolino and Alderucci [16] presented a method to evaluate the evolutionary frequency response function of classically damped linear structural systems subjected to both separable and non-separable non-stationary excitations. Chakraborty and Basu [17] proposed an input-output relation for the non-stationary response of long-span bridges subjected to random differential support motions. Their developed methodology could evaluate non-stationarity in both the intensity and frequency content of the response statistics for spatially correlated multipoint random excitations.

Random vibration analysis of structures considering the nonlinearity of the structures have also received much attention. So far, many different approaches have been developed for nonlinear random vibration analysis, each of them having their own advantages and disadvantages. Among these technique, one can mention Markov vector approach, perturbation methods, equivalent linearization, stochastic linearization, equivalent nonlinearization, closure approximations, stochastic averaging and Mont Carlo simulation method [18–20]. The most interested approaches used for nonlinear random vibration analysis are based on the linearization of the nonlinear system. For example, Feldman [21] proposed a nonlinear technique based on the Hilbert transform for investigation of nonlinear systems. His suggested approach which involves some kind of linearization of the nonlinear system, enables direct extraction of linear and nonlinear system parameters from a measured time signal of input and output. His proposed strategy was mainly developed for deterministic nonlinear vibration analysis. However, he claimed that it can extract the instantaneous modal parameters of the equivalent linear system even if the excitation is a random signal. Fujimura and Kiureghian [19] developed a new, non-parametric linearization method for nonlinear random vibration analysis. They verified the accuracy of their technique via Mont Carlo simulations. Mishra et al. [22] studied the optimum performance of the shape memory–alloy-based rubber bearing for isolating the bridge deck against a random earthquake. The responses required for modeling the nonlinear random system were obtained by stochastic linearization of the cyclic nonlinear force-deformation behavior of the shape memory–alloy restrainers. Gur et al. [23] proposed the optimal parameters for the super-elastic damper by conducting systematic design optimization, in which, the stochastic response served as the objective function, evaluated through nonlinear random vibration analysis. They assumed the response processes to be Markovian and adopted linearization technique for nonlinear force-deformation hysteresis of the building frame and the dampers.

From the provided brief literature review, one can conclude that vibration modeling of buildings under seismic activity of the ground have been well presented. However, some researchers have ignored the random nature of the earthquake excitation. Those who considered the stochastic character of the earth tremor, utilized a lumped parameter model for the multi-floor building. In practice, however, a building is a multi-body distributed parameter system constituted from some walls (which can be modeled using simple beams) interconnecting to some floors (which can be modeled using some rigid masses). As far as the authors know, such a model have not been yet reported in the literature. Accordingly, the distinct objectives and contributions of this paper are:

1. Proposing a novel multi-body distributed parameter model for a multi-floor building and deriving closed-form expressions for the natural frequencies and mode shapes of the system.
2. Analytical modeling of the random vibration response of the system to a band limited white noise excitation as the acceleration of the supporting ground.

To achieve these, Hamilton's principle is utilized to derive the partial differential equations governing the system's dynamic behavior. The normalized homogenous un-damped form of these equations are then solved for finding the natural frequencies and mode shapes of the system which are verified via finite element simulations. The suggested mode shapes are then utilized in an energy based approach to derive the temporal equations which are then modeled based on the random vibration theory and closed form expressions are derived for the statistical specifications of the response.

2. Mathematical modeling

The physical model of sample building with five floors is depicted in Fig. 1. As illustrated in this figure, each floor is considered as a concentrated line mass, while the walls are modeled as two beams, supporting the corresponded floor from the right and left. The structural damping of the system is taken into account via some concentrated dampers, which resist against the relative movements of the floors. The motion of the ground due to earthquake excitation is modeled with $\hat{\ddot{u}}_g$ and the resulting relative displacements in the structure is assumed to be small. It has to be noted that this assumption may not come true if the building experiences relatively large displacements. A distributed parameter model of this linear multi-body continuous system can be obtained using the Hamilton's principle.

Having long and slender geometry, uniform-thickness planar beams may be modeled using the Euler-Bernoulli beam theory. This theory assumes that plane cross-sections continue to remain plane and normal to the neutral axis after deformation [24] and has been successfully utilized to study the static, dynamic, and vibrational behavior of structures constituted from beams [25–27]. Assuming the Euler-Bernoulli assumptions hold for the problem under study, the strain energy stored in beams of Fig. 1 for the case of relatively small displacements can be expressed as [24]

$$\hat{\pi} = \sum_{i=1}^{5} 2\times\frac{1}{2}\int_0^{l_i} E_i I_i \left(\frac{\partial^2 \hat{w}_i(\hat{x}_i,\hat{t})}{\partial \hat{x}_i^{2}}\right)^2 d\hat{x}_i \tag{1}$$

where $\hat{\pi}$ is the strain energy of the system and l_i, E_i, I_i and $\hat{w}_i$ are respectively the length, Young's modulus of elasticity, second area