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Jacobsen's equivalent damping concept revisited

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ABSTRACT

The equivalent damping concept originally proposed by Lydik Jacobsen in 1930 is revisited in order to be of use in the seismic analysis of building structures. After briefly recalling that the equivalent damping is obtained by a linearization of a frequency response transfer function and can be deformation-dependent, the paper is then focused on its potential application. In particular, the equivalent damping is employed a) to estimate the maximum seismic displacements of first-mode dominant structures, b) in linear modal analysis, c) in response spectrum analysis and d) in energy-based considerations. Examples are provided highlighting the aforementioned applications of the equivalent damping concept, aiming to retrieve the interest of the engineering community.

1. Introduction

The equivalent linearization constitutes the most popular approximate method in earthquake engineering mainly because of its simplicity and of its ease of implementation in praxis by engineers. Traditionally, according to the equivalent linearization method, an equivalent linear structure with effective (equivalent) stiffness (period) and damping properties replaces the real non-linear structure. Both the capacity-spectrum [1] and the displacement-based seismic design [2] procedures are representative examples of the use of equivalent linearization in estimating the seismic response of structures without needing to perform non-linear time-history seismic analysis.

Evidence of the first equivalent linearization procedure, using only equivalent damping, is traced back to the original work of Lydik Jacobsen's [3,4] in the field of forced vibrations of mechanical structures. In fact, it is demonstrated in [3,4] that an equivalent damping can be found regardless of the character of the damping forces, i.e., of viscous or of frictional type. Most importantly in [3,4], the need to establish different equivalent damping for specific deformation (due to the vibration amplitude) is implied. However, in 1960, Jacobsen dealing with composite structures, due to large frequency shifts observed to joints that gradually soften or stiffen, modified his original method [5] and along with the works of Caughey [6], Rosenblueth and Herrera [7] and Jennings [8], the linearization concept using equivalent period and equivalent damping dominated [9–12] and linearization criteria were established [13]. Recent studies on the equivalent linearization concept can be also found in literature, e.g., [14,15].

Being an approximate procedure, the equivalent linearization has been several times criticized regarding its efficiency and validity, e.g., [16,17] among others. According to the opinion of the author the main reason of the criticism of the equivalent linearization is the definition of its two effective (equivalent) parameters, i.e., stiffness (period) and damping. More specifically, Makris and Kampas [16] provide concise evidence against the use of the effective (equivalent) period because its values strongly depend in the methodology used to obtain them. On the other hand, Kwan and Billington [17] provide the limitations of the several models associated with the quantification of equivalent period and equivalent damping, proposing their modifications. In [17], focusing on the underlying assumptions to obtain equivalent damping, one can realize that these assumptions cannot be typically met, rendering, thus, problematic if not erroneous the widespread approach of employing the area under a typical force-displacement plot in order to evaluate equivalent damping [18].

In an effort to revise the basis of the equivalent linearization method, it is decided to abandon the use of the equivalent period and to focus only on the equivalent damping as in [3,4]. This revised equivalent linearization method should, in a wider sense, cover any realistic multi-degree-of-freedom (MDOF) conventional (without having energy dissipation devices) building structure. To accomplish that, a double role is assigned to the equivalent damping: it has to be modal and it has to be given as function of deformation. This way these deformation-dependent equivalent modal damping ratios may indirectly account for a) the change of the dynamic characteristics (periods) and b) the effects of non-linearites (material and geometrical

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ones), both present in a seismically excited MDOF non-linear building structure.

Therefore, in the equivalent linearization using only damping procedure developed by Papagiannopoulos and Beskos [19], a MDOF equivalent linear building structure substitutes the actual MDOF non-linear one. In particular, the MDOF equivalent linear structure has the same mass and stiffness properties with the MDOF non-linear one and time-invariant equivalent modal damping ratios that take into account the effects of all non-linearities (material and geometrical). These equivalent modal damping ratios are derived with the aid of a modal damping identification model that involves a piecewise linearization of frequency response roof-to-basement transfer function [19]. To satisfy specific seismic design requirements, the equivalent modal damping ratios in [19] are given as functions of predefined deformation levels, i.e., in terms of interstorey drift (IDR).

After performing a brief discussion on the calculation of the deformation – dependent equivalent modal damping ratios, the rest of the paper is focused on the application of the equivalent damping concept. In this direction, examples are provided in order to demonstrate the use of equivalent damping in: a) the estimation of maximum seismic displacements of first mode-dominated structures, b) linear modal analysis, c) response spectrum analysis and d) energy-based considerations. On the basis of these examples, the establishment of the equivalent damping concept in earthquake engineering is judged.

2. Calculation of deformation – dependent equivalent modal damping ratios

On the basis of the results presented by Papagiannopoulos and Beskos [20], a linear building structure exhibits a smooth roof-to-basement frequency response transfer function modulus with well-defined visible peaks, which essentially correspond to its resonant frequencies. As shown in detail in [20], modal damping ratios can be calculated by using these resonant frequencies along with their corresponding transfer function moduli as well as the participation factors of the structure obtained by modal analysis.

The concept of the frequency response transfer function, originally defined for linear systems [21], can also be extended to non-linear systems [22–25]. In that case, the frequency response transfer function modulus loses its smoothness exhibiting multiple peaks or a jagged (distorted) shape. Papagiannopoulos and Beskos [19], starting from the non-smooth frequency response transfer function of a non-linear MDOF structure, achieve its eventual smoothness, indicating an equivalent linear MDOF structure, through the addition of linear viscous damping. This smoothness is performed by a per mode piecewise linearization, in which the frequency response transfer function and its derivative are checked against satisfaction of certain criteria [19]. When all modes satisfy this criteria, a set of non-linear algebraic equations of the form of Eq. (1) is numerically solved in order to obtain the modal damping ratios ξ_j of the equivalent linear MDOF structure, on the assumption that $|R(\omega = \omega_k)|$, ϕ_{rj}, ω_k and Γ_j are known or can be measured.

In Eq. (1), $|R(\omega = \omega_k)|$ is the modulus of the roof-to-basement acceleration response transfer function evaluated at the resonant frequency ω_k, ϕ_{rj} and Γ_j are the roof coordinate and participation factor of the mode j, respectively, whereas $k = 1, 2, \ldots$ N, with N being the number of resonant frequencies. Eq. (1) constitutes a modal damping identification model that holds on the assumption that the structure possess classical normal modes [26]. This damping identification model may also handle with sufficient accuracy cases of non-classical modes [27,28].

Even though it is numerically possible by solving Eq. (1) to achieve very high values for equivalent modal damping ratios, it should be checked if these values are physically acceptable considering the deformability states of the structure. In [19], by assuming the IDR as an index of the deformability state of the structure and by assigning to it a value of interest, e.g., 1.5%, one may define equivalent modal damping ratios that account for all deformations up to IDR = 1.5%. The only difference lies on the fact that the roof-to-basement acceleration frequency response transfer function should be constructed by considering the first time violation of the IDR of interest and then transform the corresponding up to that violation roof and basement acceleration-time signals in the frequency domain. The aforementioned per mode piecewise linearization is then performed, leading to equivalent modal damping ratios that account for a specific deformation level. It will be later recalled for the equivalent modal damping ratios that it is exactly this dependency on deformation that makes any need of seismic displacements determination unnecessary. Nevertheless, important at this point is to stress that the equivalent modal damping ratios computed following the aforementioned procedure, no matter which is their deformation dependency, always lead to an equivalent structure just at its first time yielding.

After performing a large number of numerical experiments involving steel framed structures subjected to various seismic motions, the end product to be of use for seismic design purposes is indicatively shown in Table 1 (taken from [19]). In this table, equivalent modal damping ratios for the first few modes of regular and orthogonal plane steel moment-resisting frames (MRFs) are shown. These equivalent modal damping ratios are provided for two types of seismic motions, i.e., near-fault ones (essentially recorded at a distance of very few kilometres from the seismic fault and are separated into two groups according to moment magnitude M_w scale) and long-duration ones (exhibiting a significant duration of over 25 s and mainly coming from earthquakes occurring at the broader area of a subduction zone). The equivalent modal damping ratios of Table 1 are also given as functions of period and for specific deformation and damage, in terms of IDR and plastic hinge rotations, respectively, levels. A dash (-) in Table 1 denotes that an equivalent damping for these modes cannot be computed. However, as mentioned in [19], to these modes an equivalent damping of almost 100% should be assigned, in order to retain accuracy in seismic response calculations.

$$|R(\omega = \omega_k)|^2 = 1 + 2\cdot \sum_{j=1}^{N} \frac{\phi_{rj}\cdot\Gamma_j\cdot\omega_k^2\cdot(\omega_j^2 - \omega_k^2)}{(\omega_j^2 - \omega_k^2)^2 + (2\cdot\xi_j\cdot\omega_j\cdot\omega_k)^2} + \sum_{j=1}^{N} \frac{\left[\phi_{rj}^2\cdot\Gamma_j^2\cdot\omega_k^4\cdot(\omega_j^2 - \omega_k^2)^2 + 4\cdot\xi_j^2\cdot\omega_j^2\cdot\omega_k^2\right]}{[(\omega_j^2 - \omega_k^2)^2 + (2\cdot\xi_j\cdot\omega_j\cdot\omega_k)^2]^2} + $$
$$+ 2\cdot \sum_{j\neq m, m>j}^{N} \frac{\phi_{rj}\cdot\Gamma_j\cdot\phi_{rm}\cdot\Gamma_m\cdot\omega_k^4\cdot[(\omega_j^2 - \omega_k^2)\cdot(\omega_m^2 - \omega_k^2) + 4\cdot\xi_j\cdot\xi_m\cdot\omega_j\cdot\omega_m\cdot\omega_k^2]}{[(\omega_j^2 - \omega_k^2)^2 + (2\cdot\xi_j\cdot\omega_j\cdot\omega_k)^2]\cdot[(\omega_m^2 - \omega_k^2)^2 + (2\cdot\xi_m\cdot\omega_m\cdot\omega_k)^2]} \quad (1)$$