Thin-Walled Structures 132 (2018) 48–57

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

# Geometrically nonlinear analysis of functionally graded shallow curved tubes in thermal environment

H. Babaei[a], Y. Kiani[b,*], M.R. Eslami[c]

[a] Mechanical Engineering Department, South Tehran Branch, Islamic Azad University, Tehran, Iran
[b] Faculty of Engineering, Shahrekord University, Shahrekord, Iran
[c] Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran

ARTICLE INFO

*Keywords:*
Shallow curved tube
Two-step perturbation technique
Temperature dependency
Functionally graded material
Thermal environment
Transverse load

ABSTRACT

Present research aims to analyze the nonlinear bending response of the functionally graded material curved tube subjected to the uniform lateral pressure. The effect of thermal environment is also included. Properties of the arch are distributed through the radial direction using a power law function. Thermomechanical properties of the media are assumed to be temperature dependent. The governing nonlinear equilibrium equations of the arch are obtained by means of the von Kármán assumption and a higher order shear deformation tube theory which satisfies the traction free boundary conditions on the inner and outer surfaces of the tube. The three coupled nonlinear equations of the tube are reduced to new two ones in a dimensionless presentation. These two equations are solved using the two step perturbation technique for pin ended and clamped ended boundary conditions. Closed form and accurate expressions are provided to estimate the deflection of the arch as a function of the thermal and mechanical load parameters. Numerical results are provided to explore the effect of different parameters such as the power law index of the FGM tube, boundary conditions of the tube, thermal environment, and three geometrical parameters.

## 1. Introduction

During the last years, many studies on the geometrically linear/nonlinear bending, buckling, free vibration, and post-buckling analysis of Functionally Graded Material (FGM) structures in thermal environments are reported in the literature. In these works, for analysis of thin-walled structures, the solutions of governing differential equations are usually solved using the approximate methods such as perturbation method, Ritz method, Galerkin method, generalized differential quadrature method (GDQM), finite element method (FEM) or other numerical techniques. A comprehensive treatment on the subject of stability of the flexural elements is given by Eslami [1,2].

Also, many studies have been made on the geometrically linear/nonlinear behavior of FGM/isotropic thin-walled structures such as beams with rectangular cross-section [3–7], beams with circular cross-section [8,9], tubes [10–13], shallow curved beams and arches [14–37], and long cylindrical panels [38–40], under several thermal/mechanical loading conditions.

Among them, Thai and Vo [3] studied the linear free vibration and bending analysis of functionally graded beams based on various higher-order shear deformation beam theories. In this study, closed-form solutions for governing differential equations using an analytical approach are obtained. Li et al. [4] studied the linear bending analysis of functionally graded Timoshenko beams with various boundary conditions subjected to arbitrary transverse loading. In this work, using an analytical approach, closed-form solutions of the Timoshenko beams are obtained in terms of the Euler-Bernoulli beams. Zhang [5] investigated the nonlinear bending analysis of shear deformable FGM beams with pinned-pinned and clamped-clamped end conditions based on the physical neutral surface concept. Zhang extracted the approximate solutions for nonlinear bending behavior of the FGM beams by using the Ritz method. Shen and Wang [6] presented a study on the geometrically nonlinear free vibration, bending, and post-buckling analysis of the FGM beams resting on elastic foundation. In this study, employing a two-step perturbation technique, approximate closed-form solutions for nonlinear governing equations are obtained. She et al. [7] studied the thermal buckling and post-buckling behavior of the FGM beams with clamped-clamped ends based on various shear deformation beam theories by using the two-step perturbation technique.

Huang and Li [8] performed an investigation on the linear buckling of FGM elastic columns with circular cross-section subjected to in-plane mechanical compressive load. In this study, the buckling analysis of

---

* Corresponding author.
*E-mail addresses:* st_h_babaei@azad.ac.ir (H. Babaei), y.kiani@eng.sku.ac.ir, y.kiani@aut.ac.ir (Y. Kiani), eslami@aut.ac.ir (M.R. Eslami).

https://doi.org/10.1016/j.tws.2018.08.008
Received 11 May 2018; Received in revised form 31 July 2018; Accepted 7 August 2018

double-walled carbon nanotubes is also considered. Moreover, Huang and Li [9] studied the linear bending and free vibration analysis of the FGM beams with circular cross-section and arbitrary radial non-homogeneity. Zhang and Fu [10] proposed a new refined beam theory for tubes with annular cross-section. In this study, displacement field of tubes is expressed as a series expansion form. Also, linear bending and free vibration analysis of tubes based on a simplified third-order beam model are presented. Zhong et al. [11] studied the nonlinear post-buckling behavior of FGM tubes with immovable pinned-pinned end conditions under thermal loading. In this work, they obtained the approximate solutions for nonlinear governing equations using a two-step perturbation technique. Moreover, Zhong et al. [12] investigated the nonlinear free vibration and bending analysis of the FGM pinned tubes resting on two parameters elastic foundation with immovable ends using a two-step perturbation technique. She et al. [13] derived approximate solutions for nonlinear bending and post-buckling response of the FGM tubes with clamped-clamped boundary conditions by employing a two-step perturbation technique.

Bradford et al. [14] studied the nonlinear instability analysis of the shallow arches under a central concentrated load. In this study, employing an analytical method, closed-form solutions for symmetric snap-through response of these structures are obtained. Pi et al. [15] analyzed the buckling of shallow arches subjected to uniform external pressure loading based on the Euler–Bernoulli arches theory using an analytical method. Pi et al. [16] investigated the non-linear buckling analysis of elastically supported circular shallow arches under uniform radial loads by an analytical method. Malekzadeh et al. [17,18] studied the in-plane/out-of-plane free vibration of the FGM circular curved beams with temperature-dependent material properties in thermal environment. In these works, employing the generalized differential quadrature method (GDQM), numerical solutions for nonlinear response of these structures are obtained. Pi and Bradford [19–21] investigated the nonlinear in-plane thermo-elastic behavior and instability analysis of shallow pinned/clamped arches under several mechanical/thermal loadings. In these works, nonlinear governing differential equations of homogeneous isotropic shallow arches are obtained based on the classical arch theory and solved by employing the analytical methods. Piovan et al. [22] studied the out-of-plane dynamic stability analysis of FGM circular arches based on two-dimensional and three-dimensional formulation models, by employing the finite element method (FEM). In the works of Stanciulescu et al. [23], Cai et al. [24], and Xenidis et al. [25] snap-through behavior of homogeneous isotropic shallow arches with various boundary conditions and subjected to thermal and mechanical loads are investigated. Wang and Liu [26] proposed an analytical elasticity solution for orthotropic functionally graded shallow circular beams under several pressure loadings on surfaces employing the Airy stress function method. Fraternali et al. [27] researched the stability analysis for the composite curved beams subjected to tensile and compressive loads. Jun et al. [28] studied the free vibration response of shallow curved beams based on trigonometric shear deformation arch theory by using an analytical method. Asgari et al. [29], Bateni and Eslami [30], and Bateni and Eslami [31] among others, studied the response of FGM arches subjected to thermal load, central connected force, and radial uniform pressure load based on the classical arches theory. Han et al. [32] investigated the nonlinear stability behavior of shallow isotropic curved beams with elastically horizontal supports. In this work, snap-through behavior of circular arches under radial uniform pressure load is studied using the principle of virtual work. Pydah and Sabale [33] studied the static behavior of bi-directional functionally graded circular curved beams based on the classical Euler-Bernoulli curved beam model. In this study, an analytical approach is employed to solve the governing differential equations. Other investigations about isotropic shallow curved beams under several load conditions with various boundary conditions can be followed in the works of Stoykov [34], Yan et al. [35], Bouras and Vrcelj [36], and Tsiatas and Babouskos [37], among others.

Nonlinear bending behavior of long FGM cylindrical panels without/with elastic foundation under uniform lateral pressure load in thermal environments are presented by Zhang [38] and She et al. [39] based on the physical neutral surface concept. Also, nonlinear thermo-elastic bending analysis of long FGM cylindrical panels on elastic foundation subjected to uniform thermal load is presented by Babaei et al. [40] using the two step perturbation technique.

As the above literature survey reveals and according to the present authors' knowledge, the nonlinear bending response of curved FGM tubes under uniform lateral pressure is not investigated so far. This research aims to fill this gap. A radially FGM tube with temperature dependent properties and immovable boundary conditions is considered. For this purpose, a higher order displacement field for the tubes and the von Kamran type of kinematic assumptions are used to construct the governing equations of the tube. These equations are solved using the two step perturbation technique and accurate closed form expressions are proposed to analyze the response of the tube under thermal and mechanical loads. Numerical results are given to explore the effects of different parameters on the geometrical nonlinear response of the curved tube.

## 2. Governing equations

Consider an elastic shallow curved tube with annular cross-section. The inner radius of cross section is $a$, outer radius is $b$, axial length is $L$, and radius of curvature is $R$. Tube is made of the FG materials from a mixture of ceramic and metal. The shallow FGM curved tube with two immovable ends is subjected to the uniform in-plane transverse distributed load and is exposed to thermal environment. Also, cross-section of the FGM shallow curved tube is referred to two polar coordinates system and Cartesian coordinates system as shown in Fig. 1.

For circular cross-section of tubes, the relationship between polar coordinates system $(r, \theta)$ and Cartesian coordinates system $(y, z)$ are expressed as

$$\begin{aligned} y &= r\cos\theta, \quad z = r\sin\theta, \\ r^2 &= y^2 + z^2, \quad a \le r \le b \quad , \quad 0 \le \theta \le 2\pi \end{aligned} \tag{1}$$

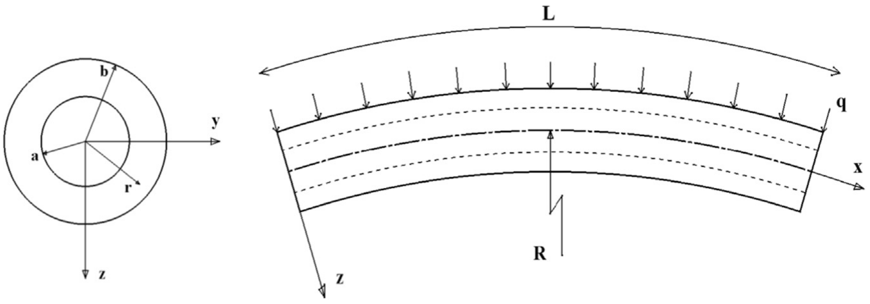

**Fig. 1.** Geometry and coordinate system of the FGM shallow curved tube. Left one: cross section of the tube, Right one: side view of the tube.