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Buckling of externally pressurised egg-shaped shells with variable and constant wall thicknesses

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ABSTRACT

This study focused on the nonlinear elastic buckling of mass equivalent egg-shaped shells with variable and constant wall thicknesses under uniform external pressure. The shells were made of photosensitive resin and had a nominal mass of 570 g, a nominal major axis of 260.36 mm, and a nominal minor axis of 181.32 mm. Four egg-shaped shells were fabricated using the aforementioned parameters and using the rapid prototype technique—stereo lithography appearance. Two shells had a constant wall thickness, whereas the remaining shells had a variable wall thickness along the meridian. The geometry of all shells was accurately measured, and the shells were slowly pressurised to destruction, thus yielding good repeatability. On the basis of the measured results, the buckling performance of the tested shells was further studied numerically. The experimental data were found to be in agreement with the numerical predictions. The results revealed that the average collapse pressure for the egg-shaped shells with variable wall thickness was approximately 24% higher than that for the egg-shaped shells with constant wall thickness, thus indicating that the load-carrying capacity of the egg-shaped shells was significantly improved when variable thickness was used.

1. Introduction

Shells of revolution subjected to uniform external pressure have received considerable attention for more than 100 years due to their high load-carrying capacity, effective inner space utilisation, and ease of fabrication [1–3]. Such shells have extensive applications in many branches of engineering, such as underwater pressure hulls, underground pressure vessels, and liquid storage tanks that are subjected to negative pressure. The typical proposed shapes of such shells are cylindrical, spherical, and conical, whereas atypical proposals include ellipsoidal, barrel-shaped, Cassini, and egg-shaped shells [2–12]. However, these shells are prone to buckling failure, which is strongly affected by the shells' meridional geometry, wall thickness, material properties, geometric imperfections, and prebuckling deformations [8–14].

A highly effective method for improving buckling capacity is to change the meridian shape; thus, numerous studies have been focused on the buckling of shells of revolution with positive Gaussian curvature. Spherical shells are the most typical example; the buckling of these shells has been widely investigated to ensure their application in deep manned submersibles [15–17] and neutrino detectors [18,19]. Moreover, the buckling capacity of spherical shells is the best compared with that of shells with other shapes because of the optimum stress and strain distribution of spherical shells. However, it is difficult to fabricate the spherical geometry because of its even and highly imperfection-sensitive character. Arranging the inner facilities of the spherical geometry is also challenging because of its constant radius curvature [20,21].

As a result, many investigators focused on the buckling of non-spherical shells of revolution with positive Gaussian curvature. For example, Blachut et al. experimentally and numerically examined the nonlinear elastic–plastic buckling of a family of mild steel barrelled shells with a circular arc and generally ellipsoidal meridians [22–24]. Moreover, Jasion and Magnucki numerically evaluated the nonlinear elastic buckling of barrelled shells with circular, Cassini, and clothoidal–spherical meridians [25–28]. On the basis of the results of Cassini oval shells revealed by Jasion and Magnucki, the nonlinear elastic buckling of externally pressurised Cassini oval shells with various shape indices were numerically and experimentally studied by Zhang et al.

More recently, from the bionic viewpoint, Zhang et al. systematically investigated the nonlinear elastic–plastic buckling of steel egg-

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shaped pressure hulls with various shape indices and wall thicknesses by using a combination of analytical, numerical, and experimental approaches [21,29–32]. An egg-shaped shell was found to have an overall superior performance compared with a spherical one. Moreover, an egg-shaped geometry was proposed by Zingoni to design sludge digesters, which provided superior sludge-mixing properties [33,34]. However, all of these shells have been considered with uniform wall thickness, and less attention has been paid to shells of revolution with variable wall thickness.

In the present study, an equivalent comparison between the buckling of mass equivalent egg-shaped shells with variable and constant wall thicknesses was conducted under uniform external pressure. First, a pair of resin egg-shaped shells with variable and constant wall thicknesses were designed and fabricated using rapid prototyping. Subsequently, the geometrical and buckling performance of the shells and the parent material properties of the shells were experimentally studied. Finally, the nonlinear elastic buckling performances of fabricated shells with measured imperfections and perfect shells with first eigenmode imperfections were numerically explored using the arc length method. The results revealed that the load-carrying capacity of egg-shaped shells is significantly improved when the shells have variable thicknesses.

2. Material and methods

2.1. Design and fabrication

Consider an externally pressurised egg-shaped shell prepared using photosensitive resin under uniform external pressure, p, with a nominal major axis (axis of revolution) $L = 260.36$ mm and a nominal minor axis $B = 181.32$ mm. Fig. 1a illustrates the radius of the prepared shell in Cartesian coordinates, which is defined as follows:

$$r_x = \pm\sqrt{L^{\frac{2}{n+1}}x^{\frac{2n}{n+1}} - x^2}, \tag{1}$$

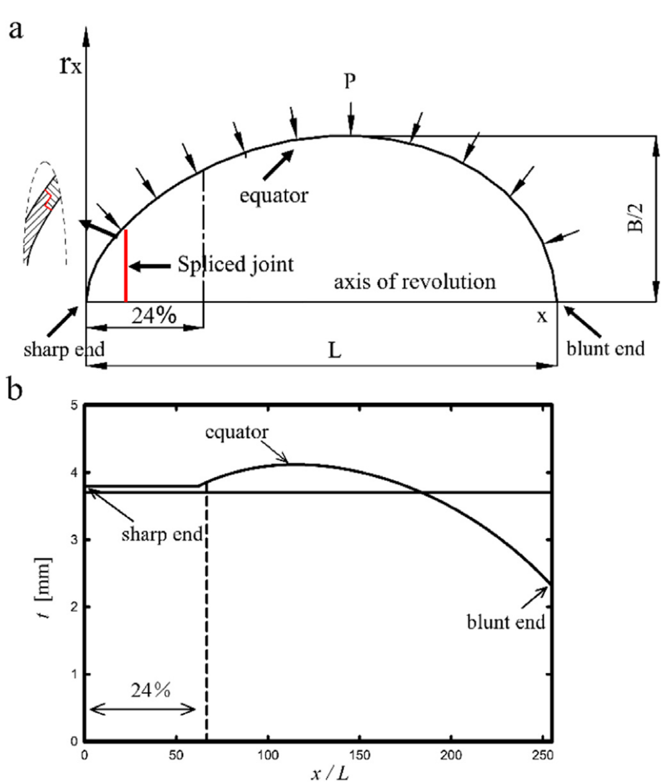

Fig. 1. (a) Schematic of an egg-shaped shell and (b) its wall thickness distribution.

where $n = 1.057(L/B)^{2.372}$. Eq. (1) was initially developed to describe the shape of eggs based on experimental results obtained from goose eggs [35,36]. Recently, the equation has been implemented to model the geometry of egg-shaped pressure hulls from the bionic viewpoint [21,29–31].

Two types of egg-shaped shells were used in this study. The first type of shell had a uniform wall thickness with a nominal value of 3.7 mm, and the second type of shell had a variable wall thickness (Fig. 1b) along the major axis with the same material volume as the first shell. A study on egg-shaped pressure hulls demonstrated that the meridional stress on these hulls is even higher than the circumferential stress [21]. In these cases, the variation in the wall thickness of the second shell is determined on the basis of the principle that the meridional stress, σ_θ, is approximately constant across the majority of the shell.

$$\sigma_\theta = \frac{p}{2t}\frac{2R_{1x}R_{2x} - (R_{2x})^2}{R_{1x}}, \tag{2}$$

Where the meridional radius of curvature R_{1x} and the circumferential radius of curvature R_{2x} are given as follows:

$$R_{1x} = -\frac{[1+(dr_x/dx)^2]^{\frac{3}{2}}}{d^2r_x/dx^2}, \tag{3}$$

$$R_{2x} = r_x\sqrt{1+(dr_x/dx)^2}. \tag{4}$$

Thus, the normalised value of the wall thickness is obtained as follows:

$$\bar{t_x} = \frac{\left|\frac{2R_{1x}R_{2x} - (R_{2x})^2}{R_{1x}}\right|}{\left|\frac{2R_{1x}R_{2x} - (R_{2x})^2}{R_{1x}}\right|_{max}}. \tag{5}$$

The derivation of the Eq. (5) was in line with Kruzelecki's claim that the instability of shells can be determined by the stress state and that buckling is initiated at the weakest point (zone) of a structure [37,38]. Moreover, because the meridional radius of an egg-shaped shell near the sharp end is relatively large, the wall thickness in this area is assumed to be constant to avoid buckling failure in the sharp zone.

Fig. 2 displays four egg-shaped shells that were fabricated upright along the axis of revolution through rapid prototyping—stereo lithography appearance (SLA). Two shells (ES1 and ES2) had constant wall thicknesses, whereas the remaining shells (ES3 and ES4) had variable wall thicknesses. All shells were fabricated using the same building parameters. Subsequently, the shells were cleaned and UV cured. The combined duration of the processes was 8 min. During fabrication, a fine point support was incorporated inside the shell to obtain an extremely smooth surface. Hand grinding was used to smooth the outer surface. Moreover, a small hole with a diameter of 30 mm was designed at the sharp end to facilitate the removal of this support. The hole was closed with a domed cap by using a cyanoacrylate adhesive ($CH_2 = C(CN) - COO{-}C_2H_5$). The cap was fabricated separately by using SLA on the basis of Fig. 1. Moreover, to ensure a strong connection between the hole and the shell, a stair-shaped margin between the components was implemented, as displayed in Fig. 1(a). As the buckling initiates far from the sharp end [30], the designed hole may have a slight effect on the buckling of shell. Photosensitive resin was selected as the fabrication material, which was adopted to study the buckling capacity of Cassini oval and spherical shells. The fabricated egg-shaped shells are illustrated in Fig. 2.

2.2. Measurement and test

Before conducting the test, the external surface of each fabricated shell was accurately measured using a three-dimensional optical scanner (Open Technologies Corporation). The accuracy of the scanner