Thin-Walled Structures 132 (2018) 167–175

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Deformation and vibration of compressed, nested, elastic rings on rigid base

Lawrence N. Virgin[a,*], Joe V. Giliberto[a], Raymond H. Plaut[b]

[a] Department of Mechanical Engineering and Materials Science, Duke University, Durham, NC 27708, USA
[b] Department of Civil and Environmental Engineering, Virginia Tech, Blacksburg, VA 24061, USA

ARTICLE INFO

Keywords:
Energy absorption
Nested rings
Nested thin-walled tubes
Lateral compression
Quasi-static loading

ABSTRACT

Vertical circular rings and a system of three nested rings are tested and analyzed. The rings are clamped to a flat rigid base and are loaded vertically at the top by either a concentrated load or a rigid plate. The tests involve rings made by 3D printing. In the analysis, each ring is modeled as an inextensible elastica. For downward quasi-static loading, point contact at the base becomes line contact under sufficiently high loads. Deformations are determined, and small vibrations about equilibrium are examined. Good agreement is found between experiments and theory. The topic is motivated by the application of short nested tubes for energy absorption.

1. Introduction

In-plane deformation and vibration of a single ring and three nested, vertical, circular rings are considered (Figs. 1 and 2). The bottoms of the rings are clamped to a horizontal, flat, rigid base, and the rings are compressed from above by a quasi-static concentrated load or horizontal rigid plate. In some cases the rings are pulled upward. Equilibria and small vibrations are investigated. Experiments are conducted, and analyses are carried out using an inextensible-elastica model for each ring. The configurations are similar to the cross sections of tubes used to absorb energy under lateral compressive loading [1–3], and the behavior of the rings is similar to that of short tubes. The elastica analysis, experimental results, and vibration results are new.

For compressive loading by a concentrated load or a plate load, a single ring has point contact with the base until a threshold value of the load is attained, and then line contact occurs. For loads smaller than this threshold load, the problem is equivalent to that of opposing concentrated loads compressing a ring. Elastica analyses of that problem have been presented in [4–10], with opposing tensile concentrated loads also included in [4,6,8,11].

Finite element analyses of a short tube compressed by a plate load are described in [12–23]. The tube material is either nonlinearly elastic or inelastic. Other types of analyses, most of which include inelastic behavior, are presented in [9,14,15,24–37]. Experiments are described in [13,14,16,18–21,23,24,29–33,38–41].

For plate loading, experiments and inelastic analyses have demonstrated that the bottom segment of the tube separates from the base (if not attached) for sufficiently high load, as does the top segment for plate loading. Similar separation has been seen for an elastic curved strip or bent elastica pushed onto a rigid base (e.g., [42,43]) when the compressive axial internal load along line contact reaches the buckling load for that segment treated as a beam with fixed ends. Such separation does not occur for the elastic rings analyzed here or for the experiments under the applied load range.

A number of papers [3,44–51] present experimental results for short, nested, circular tubes similar to the rings shown in Fig. 2 (with initial gaps between the tops of the unloaded tubes), primarily with regard to energy absorption. The results are compared to those from finite element analyses in [44–46,48–51]. Unless otherwise mentioned in the following two paragraphs, plate loads are applied, the plate speed is low (quasi-static loading), and the bottoms of the tubes are not clamped (attached) to the base or to each other at their lowest points.

In [44–46], three nested tubes are compressed experimentally. In [44], the plate moves slowly at either 0.05 mm/s or 8.3 mm/s. In [45], an indenter is also applied, modeling a concentrated load. In [47], configurations with two rings and with three rings are tested. In [48,49], the nested rings are clamped at the base, and the plate is dropped on the outer tube. Plate speeds at impact are 3–5 m/s in [48] for a three-tube system, and in [49] they are 4 m/s for a two-tube system and 4.5 m/s for a three-tube system. In [50], two nested tubes are tested, along with a single tube. In [51], a dumbbell configuration is used, with nested tubes at each end of the dumbbell. A survey of some of the experiments in these references is presented in [3].

Variations of the basic problem have also been considered. They include noncircular shapes of tube cross sections [3,47,52,53], small solid bars placed in the gaps between the tops of the tubes [3,48,47], one inner tube on top of the other inner tube [3,49], one inner tube with its axis perpendicular to those of the other tubes [44,47], the

* Corresponding author.
E-mail address: l.virgin@duke.edu (L.N. Virgin).

https://doi.org/10.1016/j.tws.2018.08.015
Received 6 April 2018; Received in revised form 14 August 2018; Accepted 16 August 2018

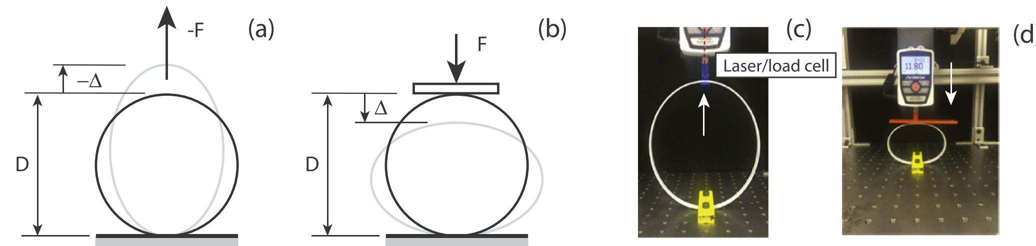

Fig. 1. Circular elastic rings: (a) pulling up with concentrated load, (b) pushing down with plate load, (c) and (d) testing configurations.

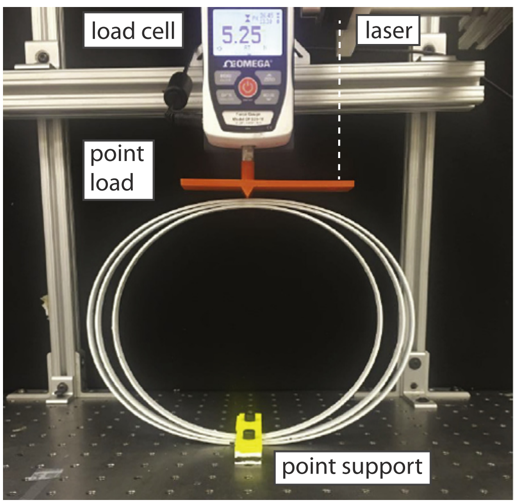

Fig. 2. A set of three nested rings subject to compressive concentrated load.

addition of external constraints on the sides of the tubes [3,45], and a V-shaped base [3,45].

The experimental set-up for the present study will be described in Section 2, and the analytical procedure will be presented in Section 3. Deformation results for a single ring will be given in Section 4, followed by results for nested rings in Section 5. Vibration frequencies for single and nested rings will be obtained in Sections 6 and 7, respectively, and concluding remarks will be given in Section 8.

2. Experimental set-up

2.1. Single ring

Slender circular rings were configured using Solidworks and printed using ABS thermoplastic with a Fortus 3D-printer. Given the general flexural stiffness dependence on EI/D^3 (where E is the modulus of elasticity, I is the moment of inertia of the cross section, and D is the diameter of the centerline), and in order to provide a reasonable range of geometric characteristics, it was decided to produce 30 individual rings according to the following rubric:

- 10-mm-wide rings: D = 100–240 mm in 10-mm increments.
- 20-mm-wide rings: D = 100–240 mm in 10-mm increments.

For all the rings, $E = 2.1$ GPa, the thickness $H = 2.15$ mm, and the density is 1040 kg/m^3.

For each ring, the bottom was clamped to the supporting table using a V-shaped device. A vertical load was applied using a displacement-controlled load cell (Omega DFG35-10). The load was applied as a concentrated load (down or up) or via a flat plate (down). Figs. 1(a) and (b) show these arrangements schematically for a concentrated load upwards and a flat plate downwards, respectively. The vertical deflection Δ (positive if downward) was measured with a proximity laser (Micro-Epsilon optoNCDT), and the load F (positive if downward) was measured with the load cell. Photographic images are shown in Fig. 1(c) and (d). Details of the vibration testing are given in Section 6.

2.2. Nested rings

For the nested rings (Fig. 2), all rings clamped together in a test had the same width. Most of the tests were conducted on rings with a 10-mm difference in centerline diameter between the outer ring and middle ring, and between the middle and inner ring. Due to the thickness of the rings, the initial vertical separation (gap) between the tops of adjacent rings was 5.7 mm for this case. For the other case of nested rings, which had a 20-mm difference in diameters, this separation was 15.7 mm. Fig. 2 shows a case with a downward concentrated load after the rings have contacted each other at the top.

3. Analysis

3.1. Single ring

Consider an unstrained, uniform, circular ring subjected to quasi-static loading (Fig. 3). The ring is modeled as an inextensible elastica, which has linearly elastic behavior and has its bending moment proportional to the difference between its deformed curvature and its initial curvature. Large deflections and rotations are allowed. The bottom of the ring is attached to a rigid horizontal base, and the top of the ring is subjected either to a vertical concentrated load (pushing down or pulling up) or a frictionless, rigid, horizontal plate (pushing down). The self-weight of the ring is neglected.

The ring has circumference $L = \pi D$, as shown in Fig. 3(a). Fig. 3(b) depicts a case of a downward concentrated load F acting at the top of the ring, and Fig. 3(c) shows a case of plate loading with total downward load F. If the load in Fig. 3(b) were upward, $F < 0$ in the analysis. For small F, the ring contacts the base at a point and the origin of the coordinate system is taken there, with arc length S, horizontal coordinate $X(S)$, vertical coordinate $Y(S)$, and angle $\theta(S)$ from the horizontal, as seen in Fig. 3(a). Beyond a threshold value of F, contact occurs along a line whose length is denoted B, and then the origin of the coordinate system is located at the right end of that line, as shown in Fig. 3(b) for a concentrated load and in Fig. 3(c) for a plate load. For plate loading, due to symmetry in the vertical direction, the contact length is B at the base and at the plate (Fig. 3(c)). If there is point contact, $B = 0$.

The ring has width W, cross-sectional area $A = WH$, bending stiffness EI, and cross-sectional moment of inertia $I = WH^3/12$. The internal force has components $P(S)$, horizontal and positive to the left on a positive face, and $Q(S)$, vertical and downward on a positive face. The bending moment is $M(S)$, positive if counter-clockwise on a positive face. The distance the force or plate moves vertically is Δ, positive if downward, as shown in Fig. 3(a). The shapes in Figs. 3(b) and (c) correspond to $F = 200EI/L^2$.

For all considered cases, the deformation is symmetric with respect to a vertical line through the top and bottom of the ring, and the analysis will be conducted on the right part of the ring above the base, in terms of the nondimensional variables