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Generating different lesson designs and analyzing their effects: The impact of representations when discerning aspects of the derivative

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ABSTRACT

This paper reports results that concern the way in which the design of instruction may influence students' opportunities to discern the relationship between a graph and its derivative graph. Two studies were conducted that together included 144 Swedish upper-secondary students who were enrolled in an introductory calculus course. In both studies, all students participated in a 120-min intervention. The first study used a qualitative approach and aimed at generating, and analyzing the outcomes of, three different lesson designs. The second study used a quantitative approach and aimed at testing the validity of the results from the first study. The results of the studies are compatible and suggest that teaching should initially focus exclusively on graphs and also include a variety of graphs. In contrast, using graphs in conjunction with formulas and/or using only graphs of polynomial functions decreased students' opportunities to discern graphical aspects of the derivative.

1. Introduction

Over the last few decades, students' understanding of the derivative has been extensively investigated (e.g. Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997; Orton, 1983; Park, 2013; Zandieh, 2000). However, while there is a rich body of research concerning students' conceptions of the derivative, there is comparatively little research about how to teach the concept. This paper reports on two intervention studies where the relation between teaching and students' learning of the derivative was investigated. In particular, the focus was on how different ways of handling the content during instruction influenced students' opportunities to discern graphical aspects of the derivative.

In previous research, there seem to be a consensus that a complete understanding of the derivative involves being able to interpret the concept in different representations (e.g. Berry & Nyman, 2003; Zandieh, 2000). Nevertheless, several studies (e.g. Asiala et al., 1997; Orton, 1983; Selden, Selden, & Mason, 1994) have reported that for many students, the derivative is primarily associated with algebraic rules and standard procedures. Already back in 1983, Orton criticized the fact that some students are introduced to differentiation as rules without reasons. Orton (1983) suggested more graphical work where the data comes from real-life situations. Similar proposals have later been repeated by several other researchers and Koirala (1997), as well as Berry and Nyman (2003), suggested initially teaching calculus through graphs and situations from real life. However, although the suggestion of more graphical work has been recurring, empirical studies of the relation between actual teaching designs and students' discernment of graphical aspects of the derivative are relatively few. One example is the study conducted by Habre and Abboud (2006). They taught a reform calculus course where concepts were introduced by the use of multiple representations. Furthermore, instead of focusing on techniques for solving drill problems, activities that allowed students to discuss problems were introduced. A study involving a reform calculus course was also conducted by Asiala et al. (1997). The instructional treatment was based on a theoretical framework and

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included three components: computer activities, classroom tasks (without computers) and exercises. The studies by Habre and Abboud (2006) and Asiala et al. (1997) involved a lot of computer work and the impact of technology has been investigated by several other researchers. For example, Ubuz (2007) examined the effect of incorporating computers in a calculus course and Berry and Nyman (2003) explored in what way graphic calculators may promote students' graphical understanding of the derivative.

In contrast to investigating the effect of teaching methods (e.g. group discussions or lectures) and technical support, the studies presented in this paper investigated how the handling of the content influenced students' discernment of graphical aspects of the derivative. Within the studies, lesson designs that aimed at offering opportunities to discern aspects of the relationship between a graph and its derivative graph were implemented. While the designs were equal with respect to teaching methods and technical support, they were different regarding the handling of the content. The handling of the content refers to which specific examples were used in the design and how the examples were sequenced. It also refers to the similarities and differences between the examples used within a design, i.e. the variation and invariance that were created through the examples. Furthermore, the handling of the content refers to which aspects of the content the teacher brought to the foreground when dealing with the examples.

The first study involved creating, implementing and evaluating the effect of three different 120-min lesson designs. In the second study, two of the designs from the first study were compared in more controlled experimental conditions. The studies addressed the following research question:

What ways of handling the content during instruction can be identified as having an impact on the improvement of students' discernment of aspects of the relationship between a graph and its derivative graph?

1.1. The need to discern graphical aspects of the derivative, some previous research results

Zandieh (2000) emphasized that students often have difficulties with transferring an interpretation of the derivative from one representation to another. Zandieh and Knapp (2006) elaborated on this further and described how one representation of the derivative may stand for the whole concept and if this is so, the student then reaches for this representation when asked to reason about the derivative, no matter what the context is.

Regarding the description of Zandieh and Knapp (2006), several studies have shown that too many students, the derivative is primarily associated with the symbolic representation. Orton (1983) interviewed 110 students and found that items concerned with understanding differentiation and graphical approaches to rate of change were difficult for the students. Items concerned with applications of differentiation turned out to be easier and Orton reported that the routine aspect of differentiation was well understood: only a few students were unable to differentiate polynomials. According to Selden et al. (1994), this preference for using algebraic techniques is also shown by more successful students. Selden et al. (1994) investigated high-performing calculus students' ability to solve non-routine problems and found that graphs seldom were used in the solutions. Similar to Orton (1983), Selden et al. (1994) argued that the students were weaker graphically than analytically and that their graphical knowledge needed improvement. The predominance of the symbolic representation of the derivative was also identified by Asiala et al. (1997), who concluded that several students from a traditionally taught calculus course were unable to work with a graphical problem without having an algebraic expression.

1.2. A specific content that offer opportunities to discern graphical aspects of the derivative

Students' preference for symbolic representation is not a new insight and several previous studies (e.g. Asiala et al., 1997; Habre & Abboud, 2006) have involved reform calculus courses aimed at developing students' graphical understanding of the derivative. However, there are also examples of studies that have focused on a particular content. Berry and Nyman (2003) conducted an observational study in which the participating students were asked to sketch antiderivative graphs. In addition to this, with the support of a graphic calculator connected to a movement detector, the students were also asked to walk the graphs as if they were displacement-time graphs. Berry and Nyman argued:

We would argue that making connections of the properties of graphs, i.e., going from graph of a function to the graph of the derived function and especially reversing the process, builds a better understanding of the underlying graphical concepts of calculus (Berry & Nyman, 2003, p. 485).

Also the current studies contained this content and it was chosen due to similar arguments as those presented by Berry and Nyman (2003). However, Berry and Nyman (2003) explored how students may "build a better understanding of graphical concepts" through the use of technology and participation in group discussions. The current studies were instead investigating which aspects of the content the students needed to discern and, furthermore, how the of handling the content during instruction influenced the opportunities. The point of departure regarding relevant aspects to focus on was previous research concerned with the same content.

Students' attempts to sketch antiderivative graphs were analyzed by Haciomeroglu, Aspinwall, and Presmeg (2010) who distinguished between analytic and visual thinking. The former typically involves translating a graph into an equation, which is integrated. This new equation is then used to sketch the graph of the antiderivative. If using a visual approach, no equation is used. Instead, the shape of the antiderivative graph is based on visual estimates of its slope. Haciomeroglu et al. (2010) concluded that the ability to synthesize these two modes of thinking is essential.

The study by Haciomeroglu et al. (2010) offered profound descriptions of students' ways of reasoning when sketching antiderivative graphs. At the same time, the interviewees were high-achieving university students and other research studies have reported how inexperienced students may struggle with more fundamental aspects. For example, Nemirovsky and Rubin (1992) determined high school students' difficulties in articulating the relationship between a function and its derivative, and found that the