Journal of Mathematical Behavior xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Supporting teachers in improving their knowledge of mathematics

Peter Sullivan

Monash University, Australia

ARTICLE INFO

Keywords:
Mathematics content knowledge
Knowledge for teaching
Pedagogical content knowledge
Challenging tasks

1. Introduction

It is self evident that teachers of mathematics are better prepared when they are aware of the potential of the mathematics they are teaching, but it is not clear how much mathematics is needed beyond the level of their teaching, what is the nature of the mathematics that teachers benefit from knowing and how they come to know mathematics they do not already know. In Australia, it is possible to graduate as a primary teacher having completed little secondary mathematics beyond year 11, and there are now substantial numbers of teachers of students in junior secondary levels who have completed little university level mathematics study. Yet research into the level of teachers' mathematics knowledge and its impact on teaching is fraught, mainly because there are both ethical and practical issues in validly assessing that knowledge. Many studies exploring this issue are either of the knowledge of prospective teachers (e.g., Henriques & Oliveira, 2013) or drawing on very small samples of practising teachers (e.g., Leikin & Zazkis, 2010). The results reported below are unique in that they compare teachers' responses to a particular mathematics question prior to teaching a lesson involving the question with those same teachers' responses afterwards.

The results are a subset of those from a larger project[1] that is examining a particular approach to teaching that arguably places demands on the mathematics knowledge of teachers. The proposition in the overall project is that students learn mathematics best if they engage in building connections between mathematical ideas for themselves prior to instruction from the teacher. The project is studying the type of tasks, termed challenging, that can be used to prompt this learning and ways that those tasks can be optimally incorporated into teachers' usual repertoires. Essentially the notion is for teachers to pose problems that the students do not yet know how to solve and to structure lessons that allow students to find solutions for themselves, part of which is encouraging students to persist with the challenges. Such an approach assumes that teachers have the mathematical knowledge to become aware of the possibilities within each task not only in advance but also while the lessons are in progress.

The data reported below are intended to offer insights into the following research question:

> *Can teachers learn aspects of mathematics through planning and teaching lessons based on suggestions about challenging mathematics tasks?*

The data are from teachers' responses to prompts associated with a particular mathematics task, termed *two purchases*, posed prior to teaching a lesson incorporating the task and after the teaching. The task was posed to the teachers as follows:

E-mail address: peter.sullivan@monash.edu.

[1] The Encouraging Persistence Maintaining Challenge project was funded through an Australian Research Council Discovery Project (DP110101027) and was a collaboration between Monash University and Australian Catholic University. The generous participation of project schools is acknowledged.

http://dx.doi.org/10.1016/j.jmathb.2017.08.006
Received 24 November 2016; Received in revised form 25 June 2017; Accepted 26 August 2017

The total cost of one of my pairs of shoes and one of my pairs of sandals is $87. I know that the shoes cost at least $50 more than the sandals. How much might the shoes cost? (give at least 5 possibilities).

2. Perspective on teacher knowledge

The overall project is informed by a theoretical model of influences on teachers, based on Clark & Peterson (1986), which proposes that teachers' actions are informed by their intentions. These intentions are, in advance, influenced by their knowledge of mathematics and associated pedagogies, their attitudes and beliefs, and opportunities and constraints teachers anticipate they might experience when teaching. The ways that the model informed the project overall is described in detail in Sullivan et al. (2014). One of the benefits of the model is that it provides a process for anticipating reasons that teachers might be reticent in incorporating challenging tasks and the associated pedagogies into their repertoires. For example, teachers may be reluctant to adopt problem based approaches if they feel that challenging tasks do not align with their own knowledge of mathematics or its pedagogy (knowledge), if they feel that students learn best by following taught procedures (beliefs), or if they fear negative reactions from students (constraints). In this article, the focus is on the mathematics content knowledge of teachers.

In articulating different types of teacher knowledge, Hill, Ball, and Schilling (2008) described diagrammatically components of two types of knowledge. The first type, subject matter knowledge, includes common content knowledge, specialised content knowledge and knowledge at the mathematical horizon. Hill et al. (2008) explained that common content knowledge includes the mathematics that an educated person might need to solve a challenging task. In the case of the *two purchases* task presented above, the knowledge includes awareness of the importance of reading the question using all of the information, and being aware that trial and error is an appropriate solution method. Hill et al. used the term specialised content knowledge to include the flexibility to interpret student solutions, to be aware of different representations of the problem and that students' emerging intuitive approaches are as much part of learning the relevant mathematics as algorithmic routines. Such knowledge informs, in particular, the reviews of student derived strategies. The term, knowledge at the mathematical horizon, includes awareness that the concepts inherent in the task are part of the mathematics curriculum at middle secondary levels and that thinking about two complementary pieces of information creating simultaneous equations is useful in a variety of practical situations. Knowledge at the horizon can provide a rationale for incorporating a challenging task into their program in the first place. Such knowledge connects with what Rowland, Huckstep, and Thwaites (2005) described as Foundation in their knowledge quartet.

The second type of knowledge described by Hill et al. (2008), pedagogical content knowledge, is also important for the data reported below because it is through the teaching that the teachers in the project engaged with the pedagogies associated facilitating student learning while and through working on challenging tasks. The key aspect is that teachers were encouraged to pose the task without instructing students on the solution methods, to observe students while they were working, and to use student solutions to prompt mathematical discussions (see Smith & Stein, 2011).

The notion of a scheme of utilisation, as proposed by Gueudet and Trouche (2011), is used to offer suggestions to the teachers about relevant pedagogical approaches. In this, specific suggestions of mathematical tasks and a pedagogical approach in which the tasks are presented as suggested lessons were offered to teachers. Rather than disempowering teachers, the notion of supporting teacher learning about pedagogy by providing specific suggestions was found to prompt reflection on teaching by Jackson and Marakrin (2016). They provided specific on-line resources to which teacher had access, which is similar in effect to this overall project.

A different perspective on prompting reflection on pedagogy was proposed by Stephan, Pugalee, Cline, and Cline (2017) who suggested that teachers should imagine lessons before teaching them. In particular, they listed the key stages of this imagining as:

Unpacking the lesson objective
Talking though how to launch the task
Anticipating how students will engage with the task and what their misconceptions might be
Deciding which strategies will be presented and in what order
Deciding what questions to ask to provoke reflection
Determining what counts as evidence that students have understood the ideas. (p.8)

The advantage of lesson imagining is that it presents the lesson as more than the task. The Stephan et al. (2017) steps assume that students will work on tasks that have more than one possible solution and more than one possible solution strategies. The steps assume that not only will students have opportunities to learn the mathematics but also that working in this way has potential to expose partial conceptions. The suggestions offered to teachers in this project also prompted teachers to consider the learning experiences holistically. In particular, Sullivan et al. (2014) summarised the information offered to teachers for each lesson in this project as including:

- one or more task(s) that are challenging and designed to prompt the building of connections between ideas which are posed with minimal instruction and which are reviewed drawing on the explorations of the students;
- one or more additional task(s) that are intended to consolidate the earlier learning;
- preliminary experiences that are pre-requisite to but which do not detract from the challenge of the tasks; and
- supplementary tasks for differentiating the experience through the use of
    - enabling prompts (see Sullivan, Mousley, & Jorgensen, 2009) which involve reducing the number of steps, simplifying the complexity of the numbers, and varying the forms of representation for those students who cannot proceed with the task;