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The general prenucleolus of n-person cooperative fuzzy games

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Abstract

In this paper, we define the concept of the general prenucleolus minimizing the player complaint vector in the lexicographic order over the preimputation set for the cooperative games with fuzzy coalitions. For the general prenucleolus, we get a sufficient condition that the complaints of all players are in equal amount under the case of linear complaint functions concluding the proposed player excess (Sakawa and Nishizaki (1994) [15]). As a result, we can obtain the proposed fuzzy solutions with the corresponding linear complaint functions, such as the equalizer solution (Molina and Tejada (2002) [5]) and Shapley function etc. We prove that the least square general prenucleolus minimizing the variance of the resulting player complaints, is the general prenucleolus. Thus, an optimal solution considered from two aspects of the lexicographic order and the least square criterion, is obtained. In addition, several general prenucleoli are proposed in terms of specific situations.

Keywords: Fuzzy coalition; General prenucleolus; Least square; Lexicographic order

1. Introduction

In classical cooperative game theory, the concepts of the prenucleolus and nucleolus minimizing the coalition excesses in the lexicographic order over the imputation and preimputation set respectively, were firstly introduced by Schmeidler [4]. When the dissatisfactions of coalitions were weighted by the size of the coalitions, this procedure generated the per capita (pre)nucleolus [9]. Alike the (pre)nucleolus and the per capita (pre)nucleolus only depending on the worths of the coalitions, the modified (pre)nucleolus [18] considering the difference of excesses as a measure of dissatisfaction to treat all coalitions equally as far as possible, was defined. Simultaneously, the shortcoming is that the computational complexity is higher than that of the (pre)nucleolus. Then Tarashnina [16] defined the simplified modified (pre)nucleolus regarding with the difference of the excesses of a pair of complementary coalitions to escape from the issue. As the idea of lexicographically minimizing (maximizing) a vector of objective functions will not applied only to the case of TU games, Mascher et al. [14] proposed the concept of general nucleolus, a generalization

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of the nucleolus to arbitrary pair (Π, F), where Π is a topological space and F is a finite vector whose components are real and continuous functions defined on Π.

When the summation of all coalition excesses is a constant, some coalition excesses will be increased, which is caused by the decrease of partial coalition excesses. Ruiz et al. [12] explored the solution concept of the least square (pre)nucleolus which is to choose a payoff vector minimizing the variance of the resulting excesses of the coalitions different from the method of the lexicographic order. More accurately, the least square (pre)nucleolus is the payoff vector whose associated excesses are closest to the average excess under the least square criterion over the (pre)imputation set. Moreover, Molina and Tejada [6] proved that the least square nucleolus is a general nucleolus. Then one may wonder whether there exists an optimal solution obtained by considering both the lexicographic order and the least square criterion.

As the solutions mentioned above are all based on a various of coalition complaints reflecting the dissatisfaction or loss of a coalition, it can not intuitively response the dissatisfaction or loss of the players themselves. Sakawa and Nishizaki [15] firstly defined the player excess to evaluate everyone's payoff by summing up all of the excesses of coalitions to which he belongs. However, Aubin [11] proposed the concept of fuzzy coalition and introduced the cooperative fuzzy game since agents may face the case of partical cooperation due to the variability of environment or the limitation of available resource rather than either all players will cooperate with each other or the game will be played noncooperatively. Identically, Sakawa and Nishizaki [15] gave the player excess in a fuzzy game.

Actually, we define the concepts of the general prenucleolus minimizing the player complaint vector in the lexicographic order and the least square general prenucleolus minimizing the variance of the resulting complaints of players over the preimputation set on the space of cooperative games with fuzzy coalitions. For the general prenucleolus, we get a sufficient condition that the complaints of all players are in equal amount if they are all of linear complaint functions, while the equalizer solution in Molina and Tejada [5] is given under the hypothesis of taking the same excess for all of players. As a result, we can obtain many proposed fuzzy solutions with the corresponding linear complaint functions. Meanwhile, the complaints of all players are the same with respect to the least square general prenucleolus under the case of linear complaint functions as well. Thus, an optimal solution considered from two aspects of the lexicographic order and the least square criterion, is obtained.

In addition, we here give several specific linear complaint functions under the condition whether the players' participation levels are given or not. Once the fuzzy coalition is given, we consider the difference of the marginal contribution and the corresponding payment to define the player excess and further get the optimal solutions under two situation whether the players with their participation levels join the cooperation under an order or not. For an arbitrary fuzzy coalition, two integral forms of the player excesses are given by treating a coalition and its complementary coalition equally and even the per capita case. Moreover, we prove that the (per capita) envy fuzzy prenucleolus of a fuzzy game with Owens multilinear extension [8] coincides with the least square prenucleolus of the corresponding crisp game, which concurrently gives us a method of computing the least square prenucleolus of a crisp game by means of the multilinear extension. At last, a system of axioms are list to axiomatically characterize the mentioned solutions.

The paper is organized as follows: In Section 2, we get some preliminary knowledge served for the later contexts and some important conclusions. Section 3 introduces the general prenucleolus with given participation level. The general prenucleolus with integral forms is proposed for Section 4. Section 5 provides the general prenucleolus with integral forms for the cooperative fuzzy games with multilinear extension form. The axiomatizations of the corresponding solutions are given in Section 6. Section 7 concludes with a brief summary.

2. Preliminaries

2.1. The general nucleolus of crisp cooperative game

A cooperative game with transferable utility (TU) is a pair (N, v), where $N = \{1, 2, \cdots, n\}$ is the set of players and $v : 2^N \to R$ is a real-valued function satisfying $v(\emptyset) = 0$ with 2^N representing the subsets of N named with coalitions. For each coalition $S \subseteq N$, $v(S)$ denotes the worth that coalition S achieves with its members cooperative altogether. $|S|$ denotes the cardinality of S and noting that $n = |N|$ for convenience. If there is no ambiguity, we identify the game (N, v) with its characteristic function v. The set of all crisp cooperative games over N is denoted by G^N.

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