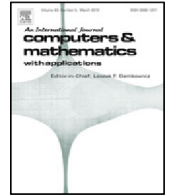




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On global bifurcation for a cross-diffusion predator–prey system with prey-taxis[☆]

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ABSTRACT

This paper is concerned with a cross-diffusion predator–prey system with prey-taxis incorporating Holling type II functional response under homogeneous Neumann boundary condition. By employing global bifurcation theory, it is obtained that a branch of nonconstant solutions can bifurcate from the positive constant solution whenever the chemotactic is attractive or repulsive. Furthermore, by using perturbation of simple eigenvalues it is found that the bifurcating solutions are locally stable near the bifurcation point under suitable conditions. These results imply that cross-diffusion can create coexistence for the predators and preys under the above special case.

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1. Introduction

In population ecology, a central topic is the spatial and temporal behaviors of interacting species in ecological systems. One aspect of great interest for a prey–predator model is whether the stationary pattern arises. Recently, stationary pattern induced by diffusion has been studied extensively (see [1–4]). Another aspect of great interest for a model with species interactions is whether the involved species can persist or even stabilize at a coexistence steady state. Many researchers have already investigated persistence, stability and bifurcation of the ecological systems with diffusion (see [5–8]).

In foraging for food and reproductive opportunity, a characteristic feature of population is that they often respond to the environment. One such response follows a tendency which moves towards or away from an external stimulus, and this tendency is facilitated by turning angle, directional behavior-change of speed and nondirectional behavior-change of velocity or turning rate in response to the stimulus [9]. In world of the predators and preys, the spatial–temporal variation of velocity for the predator is always directed by prey gradient, which is called prey taxis. Therefore, prey taxis means that the movement of predator is always controlled by prey density. The mechanism of prey-taxis was supported by some studies for measuring characteristics of individual tendency [10,11]. Thereafter the prey-taxis system was first formulated by paper [12] and was extended by [13]. In paper [14], the authors considered the spatial predator–prey systems with prey-taxis. Particularly, they investigated the foraging behavior of the predator towards high prey density and studied the role of prey-taxis in spatial predator–prey interactions.

Modeling spatial segregation phenomena of species in population dynamics, Shigesada, Kawasaki and Teramoto proposed cross-diffusion model [15] in 1979. Cross-diffusion expresses the flux of species due to mutual interferences. The value of the cross-diffusion coefficient may be positive and negative. The species moves towards lower concentration of another species when the cross-diffusion coefficient is positive and the population tends to higher concentration of another population when

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the cross-diffusion coefficient is negative [16,17]. In paper [18], a general cross-diffusion population system was put forward by numerical simulating. For more detailed biological backgrounds of this kind of model, one can also refer to [19,9].

In this paper, we are interested in the following cross-diffusion predator–prey model with prey-taxis:

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u - \nabla \cdot (\chi u \nabla v) - au + \frac{ecu v}{1 + bv} & \text{in } \Omega \times (0, T), \\ \frac{\partial v}{\partial t} = \Delta(v + duv) + r\left(1 - \frac{v}{K}\right)v - \frac{cuv}{1 + bv} & \text{in } \Omega \times (0, T), \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega \times (0, T), \\ (u(0, x), v(0, x)) = (u_0(x), v_0(x)) \geq (0, 0) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $R^N (N \geq 1$ is an integer) with a smooth boundary $\partial\Omega$; u and v represent the densities of the predator and prey respectively; χ denotes the prey-tactic sensitivity. The term $\chi u \nabla v$ gives the velocity by which predators move up the gradient of prey. For more detailed meaning of other coefficients, one can refer to [20].

When $d = 0$, there are some papers for considering the system (1.1). In [21], the authors studied the existence of weak solutions by using the Schauder fixed-point theorem, and the uniqueness of solution via the duality technique. In [22], the author analyzed the classical solution in $C^{2+\sigma, 1+\sigma/2} \times C^{2+\sigma, 1+\sigma/2} (0 < \sigma < 1)$. Li et al. studied the existence and stability of steady-state solutions [20]. He et al. proved global boundedness of solutions [23]. In paper [14], the author investigated the traveling waves to understand how the movement of predator could be controlled by the behavior of prey. Wang et al. proved that a branch of nonconstant solutions can bifurcate from the positive equilibrium only when the chemotactic is repulsive and found the stable bifurcating solutions near the bifurcation point under suitable conditions [24]. For the study of the cross-diffusion population system, one can refer to [20,25,19].

The corresponding elliptic system of (1.1) can be expressed as:

$$\begin{cases} \Delta u - \nabla \cdot (\chi u \nabla v) - au + \frac{ecu v}{1 + bv} = 0 & \text{in } \Omega, \\ \Delta(v + duv) + rv\left(1 - \frac{v}{K}\right) - \frac{cuv}{1 + bv} = 0 & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.2}$$

According to Theorem 2.2 in [24] and Theorem 2.3 in [20], there do not exist spatial patterns (nonconstant positive solutions) for system (1.1) with $d = 0$ when the chemotactic χ is attractive (that is $\chi > 0$) under some conditions. Then authors studied a branch of nonconstant solutions from the positive constant solution for system (1.1) without cross-diffusion only when the chemotactic χ is repulsive (that is $\chi < 0$) [24]. For this paper, we shall investigate the effect of cross-diffusion coefficient d on arising pattern formation for system (1.1). The main objective is to find a branch of nonconstant solutions bifurcating from the positive constant solution even if the chemotactic χ is attractive (that is $\chi > 0$) with $d \neq 0$ for system (1.1). Motivated by papers [24] and [26], we employ another version of the well-known Crandall–Rabinowitz’s bifurcation theory developed by paper [26] to investigate the system (1.1). When $d \neq 0$, the cross-diffusion of the prey species impacts on the dynamic of the system (1.1). Moreover, with $d \neq 0$, a priori estimate $\|u\|_{L^\infty}$ becomes more difficulty than that of [24] and the proof of the local stability of the bifurcation branches is also more complicated.

For convenience of discussion, throughout this paper, we always assume that $\chi > 0$ and $d > 0$.

This paper is organized as follows. In Section 2, we present the preliminaries. In Section 3, we make a priori estimates of (1.1). In Section 4, global bifurcation of (1.1) is investigated. In Section 5, we analyze local stability of the bifurcation branches. In Section 6, the comments and conclusions are presented.

2. Preliminaries

There exist three non-negative constant solutions $(0, 0), (0, K)$ and $w_* = (u_*, v_*)$ for system (1.1), where

$$u_* = \frac{re}{(ec - ab)} \left(1 - \frac{a}{K(ec - ab)}\right), \quad v_* = \frac{a}{ec - ab}.$$

For ensuring the existence of a positive constant solution of (1.1), it is necessary to assume that the inequality $K(ec - ab) > a$ holds throughout this paper.

Denote

$$\begin{aligned} Tr(M(\mu, \chi)) &= -2\mu + \left(r - \frac{2rv_*}{K} - \frac{cu_*}{(1 + bv_*)^2}\right), \\ det(M(\mu, \chi)) &= \mu^2 + \left(-r + \frac{2rv_*}{K} + \frac{cu_*}{(1 + bv_*)^2} + \frac{cu_* v_*}{1 + bv_*} \chi\right)\mu + \frac{ec^2 u_* v_*}{(1 + bv_*)^3}. \end{aligned}$$

The following Lemmas state the results of stability for (u_*, v_*) of (1.1) when $\chi > 0$ and $d = 0$.

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