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Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Multiple rogue wave and breather solutions for the (3+1)-dimensional KPI equation

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ARTICLE INFO

Article history:
Received 13 October 2017
Received in revised form 3 May 2018
Accepted 2 June 2018
Available online xxxx

Keywords:
Rogue wave
Breather
Kadomtsev–Petviashvili equation
Symbolic computation

ABSTRACT

By a symbolic computation approach, rouge wave solutions with controllable center of the (3+1)-dimensional Kadomtsev–Petviashvili I equation (KPI) can be presented. The first and the second order rouge wave will be obtained. Applying to Hirota bilinear method, we obtain multiple breather solutions and interaction solutions for the (3+1)-dimensional KPI equation.

1. Introduction

Rouge wave was discovered early on the ocean because of its giant destructive power and it cannot be predicted [1]. As another nonlinear science revolution after soliton, the study of rouge wave has drawn much attention of researchers and risen in many fields such as nonlinear optics [2–4], oceanic [5–8], financial rouge waves [9], Bose–Einstein condensate [10,11], and water wave tank [12]. In 1983, D. H. Peregrine has obtained several analytical solutions of NLS equation [13], then the first-order and higher-order rouge wave solutions of NLS equation [14,15], rouge waves solutions of variable coefficient NLS equation [16], coupled NLS equation [17], and derivative NLS equation [18] have been obtained in succession. Rouge waves of the (2+1)-dimensional Davey–Stewartson (DS) II equation [19] and Kadomtsev–Petviashvili (KP) equation [20,21] have been obtained. We will consider (3+1)-dimensional KPI equation [22]. It is usually written as

$$(u_t + 6uu_x + u_{xxx})_x - 3u_{yy} - 3u_{zz} = 0. \tag{1}$$

At present, there are several approaches to solve partial differential equations [23–27] and a class of lump solutions of KP equation has been given [28]. The rouge wave solutions with controllable center of the (2+1)-dimensional KP equation have been given by applying a symbolic computation approach [20]. The solitons and dromions of the (3+1)-dimensional KP equation with real and complex parameters have been obtained [29]. Extending real parameters to complex parameters, multiple breather solutions and the significant interaction solutions for (2+1)-dimensional KP equation have been presented [30].

The paper is organized as follows. In Section 2, we present the first and second order rouge wave solutions with controllable center for (3+1)-dimensional KPI equation. In Section 3, we give multiple breather solutions and the interaction solutions for (3+1)-dimensional KPI equation.

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https://doi.org/10.1016/j.camwa.2018.06.001

2. Rogue wave solutions with a controllable center

According to the symbolic computation approach [20,31–33], we obtain rogue wave solutions with a controllable center for (3+1)-dimensional KPI equation.

Setting $\xi = x + by - ct$ in Eq. (1) gives

$$(-c - 3b^2)u_{\xi\xi} + 6(uu_\xi)_\xi + u_{\xi\xi\xi\xi} - 3u_{zz} = 0. \tag{2}$$

Under the transformation

$$u = u_0 + 2(\ln f)_{\xi\xi}, \tag{3}$$

Eq. (2) becomes the following Hirota bilinear equation

$$\begin{aligned}&(D_\xi^4 - \omega D_\xi^2 + 3\sigma^2 D_z^2)f \cdot f\\ &\quad = 2\left[ff_{4\xi} - 4f_\xi f_{3\xi} + 3f_{2\xi}^2 - \omega(f_{2\xi}f - f_\xi^2) + 3\sigma^2(f_{2z}f - f_z^2)\right] = 0,\end{aligned} \tag{4}$$

where $\omega = c + 3b^2 - 6u_0$. We choose f as

$$\begin{aligned}f \ &= a_{6,0}\xi^6 + (a_{4,0} + a_{4,2}z^2)\xi^4 + (a_{2,0} + a_{2,2}z^2 + a_{2,4}z^4)\xi^2 + (a_{0,0} + a_{0,2}z^2 + a_{0,4}z^4\\ &\quad + a_{0,6}z^6) + 2\alpha z(a_{0,1} + a_{0,3}z^2 + a_{2,1}\xi^2) + 2\beta(a_{1,0} + a_{1,2}z^2 + a_{3,0}\xi^2)\xi.\end{aligned} \tag{5}$$

Substituting (5) into (4) and setting all the coefficients of the different powers of $z^p\xi^q$ $(0 \le p+q \le 10)$ to zero, we obtain a system of 44 polynomial equations, which are listed in the Appendix. With the help of the symbolic computation system Maple or Mathematica, solving and omitting some cases which lead to trivial and singular solutions of Eq. (1), we have

Case 1.

$$\begin{aligned}&a_{0,0} = \frac{3\alpha^2 a_{0,1}^2 + \beta^2\omega a_{1,0}^2 + 3a_{2,0}^2}{\omega a_{2,0}}, \quad a_{0,2} = \frac{\omega a_{2,0}}{3},\\ &a_{0,4} = a_{0,6} = a_{2,2} = a_{2,4} = a_{4,0} = a_{4,2} = a_{6,0} = a_{0,3} = a_{2,1} = a_{1,2} = a_{3,0} = 0;\end{aligned} \tag{6}$$

Case 2.

$$\begin{aligned}&a_{0,0} = -\frac{3\alpha^2 a_{0,1}^2 + \beta^2\omega a_{1,0}^2 + 3a_{2,0}^2}{5\omega a_{2,0}}, \quad a_{0,2} = -\frac{9\omega a_{2,0}}{15}, \quad a_{0,4} = -\frac{17\omega^3 a_{2,0}}{1125},\\ &a_{0,6} = -\frac{\omega^5 a_{2,0}}{3375}, \quad a_{2,1} = \frac{3\omega a_{0,1}}{5}, \quad a_{2,2} = -\frac{6\omega^2 a_{2,0}}{25}, \quad a_{2,4} = -\frac{\omega^4 a_{2,0}}{375}, \quad a_{4,0} = -\frac{\omega a_{2,0}}{5},\\ &a_{4,2} = -\frac{\omega^3 a_{2,0}}{125}, \quad a_{6,0} = -\frac{\omega^2 a_{2,0}}{125}, \quad a_{0,3} = -\frac{\omega^2 a_{0,1}}{15}, \quad a_{1,2} = \omega^2 a_{1,0}, \quad a_{3,0} = -\omega a_{1,0},\end{aligned} \tag{7}$$

where $\omega = c + 3b^2 - 6u_0$.

Corresponding to the above solutions, we find the following rational solutions of Eq. (1):

Case 1.

$$u = u_0 + \frac{12\omega a_{2,0}^2 \Delta_-}{\Delta_+^2}, \tag{8}$$

where $\omega = c + 3b^2 - 6u_0$,

$$\Delta_+ = 9\alpha^2 a_{0,1}^2 + 6z\alpha\omega a_{0,1}a_{2,0} + (9 + 3\xi^2\omega + z^2\omega)a_{2,0}^2 + 3\beta^2\omega a_{1,0}^2 + 6\beta\xi\omega a_{1,0}a_{2,0},$$

$$\Delta_- = 9\alpha^2 a_{0,1}^2 + 6z\alpha\omega a_{0,1}a_{2,0} + (9 - 3\xi^2\omega + z^2\omega)a_{2,0}^2 - 3\beta^2\omega a_{1,0}^2 - 6\beta\xi\omega a_{1,0}a_{2,0}.$$

The first order rogue wave solutions of KPI equation are given in Figs. 1 and 2. For Fig. 1. the parameters are selected with $\alpha = \beta = 0, b = a_{0,1} = a_{1,0} = a_{2,0} = 1, c = u_0 = -1$. We find that the center of the rogue wave is at $(0, 0)$. This solution is described respectively at (ξ, z), (z, t) and (y, t), and the solution is a line-soliton at (y, t). For Fig. 2. the parameters are chosen with $\alpha = 3, \beta = 2, b = a_{0,1} = a_{1,0} = a_{2,0} = 1, c = u_0 = -1$. In this case, the center of the rogue wave is at $(-2, -\frac{9}{8})$, This solution is described respectively at (ξ, z), (z, t) and (y, t) and the solution is a line-soliton at (y, t).

Case 2.

$$u = u_0 + 2\left(-\frac{f_x^2}{f^2} + \frac{f_{xx}}{f}\right), \tag{9}$$