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Traveling wave solutions for Gause type predator–prey systems with density dependence: A heteroclinic orbit in $\mathbf{R}^4$ ☆

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ABSTRACT

This paper investigates the existence of traveling wave solutions for a class of diffusive Gause type predator–prey systems with linear density-dependence for predator. Using the shooting method, we establish the existence of a non-negative traveling wave solution connecting a boundary equilibrium point to the coexistence equilibrium point with the help of topological method of Wazewski Theorem and LaSalle's invariance principle.

1. Introduction

In this paper, the existence of traveling wave solutions for the following diffusive Gause type predator–prey system with linear density-dependence for predator

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \Delta u(x,t) + B(u(x,t)) - f(u(x,t))w(x,t),\ x \in \Omega \subset \mathbf{R},\ t > 0, \\ \dfrac{\partial w(x,t)}{\partial t} = d_2 \Delta u(x,t) + \alpha w(x,t) f(u(x,t)) - (\beta w(x,t) + \sigma w^2(x,t)),\ x \in \Omega \subset \mathbf{R},\ t > 0 \end{cases} \tag{1}$$

is considered, where Ω is a one-dimension spatial domain for mathematical and conceptual simplicity, Δ is the Laplace differential operator, α, β and σ are positive constants, $d_1 > 0$ and $d_2 > 0$ are diffusion coefficients, $u(x,t)$ and $w(x,t)$ are the population densities of the prey and predator at time t and location x for prey and predator respectively, $B(u)$ and $-(\beta w + \sigma w^2)$ are the birth function both with density dependence for prey and predator respectively, $f(u)$ is the functional response which describes consumption rate of prey by a unit number of predator. $B(u)$ and $f(u)$ are the smooth functions satisfying some conditions which will be specified below.

Dynamical systems generated by predator–prey models have long been an important subject of research interest of many theoretical and empirical ecologists and mathematical scientists, and there has been a vast literature to investigate the dynamics of predator–prey models, see e.g., references [1–24] and references therein. As it was said in [15], space and spatial features are now solidly established as essential considerations in ecology both in terms of theory and practice, and the mathematical challenges in advancing understanding of the role of space in ecology are substantial and mathematically

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seductive. If space and spatial features are caused by diffusion movements of the concerned species, the general class of the predator–prey models take the following form of Kolmogorov type

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \Delta u(x,t) + u(x,t) f(u(x,t), v(x,t)), \ x \in \Omega, \ t > 0, \\ \dfrac{\partial v(x,t)}{\partial t} = d_2 \Delta v(x,t) + v(x,t) g(u(x,t), v(x,t)), \ x \in \Omega, \ t > 0, \end{cases} \tag{2}$$

where $\Omega \subset \mathbf{R}^n$ is the habitat, $f(u,v)$ and $g(u,v)$ are usually the functions of class C^1 satisfying the properties $\frac{\partial f}{\partial v} < 0$ and $\frac{\partial g}{\partial u} > 0$, respectively, in view of the biological significance.

It is well known that the bounded traveling wave solutions can be characterized as the solutions invariant with respect to translation in space and describe transition processes. Transition from one equilibrium to another one is so typical that the existence of it becomes an important topic for parabolic equations which models many physical, chemical and biological phenomena, see monograph [25–27], survey [28], textbook [29] and references therein for more details. Dunbar [30–32] is the pioneer to consider the existence of traveling wave solutions for diffusive predator–prey models. In [30,31], the author first investigated the traveling wave solutions connecting one equilibrium point to another one of diffusive Lotka–Volterra models with the diffusion coefficients for prey $d_1 = 0$ and $d_1 \neq 0$, respectively, and then, in [32] the author further considered the traveling wave solutions connecting an equilibrium point to a periodic orbit of diffusive Holling type II predator–prey models with the diffusion coefficients for prey $d_1 = 0$. Also, Gardner [33] studied traveling wave solutions of predator–prey systems using a connection index argument, and then, Mischaikow and Reineck [34], using the Conley index, continuation arguments and connection matrix theory, established existence of various types of traveling waves of abstract predator–prey systems. Later, Huang, Lu and Ruan [35] further established traveling wave solutions connecting two equilibria and small amplitude traveling wave train solutions of Holling type II predator–prey model with $d_1 \neq 0$. Li and Wu [36] considered Holling type III predator–prey systems with $d_1 = 0$, they also established the similar results as in [35]. The method of the proof employed in [35,36] is the shooting argument used in [30,31] and Hopf bifurcation theorem. Recently, many authors continued to study traveling wave solutions of predator–prey systems, such as, Lin, Wu, and Weng [37] generalized the results of [36] to more general systems, and then, Hsu and Yang et al. [38] and Huang [39] investigated the following predator–prey systems with more general birth functions and functional responses

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \Delta u(x,t) - h(u(x,t))[g(v(x,t)) - p(u(x,t))], \ x \in \Omega, \ t > 0, \\ \dfrac{\partial v(x,t)}{\partial t} = d_2 \Delta v(x,t) - \ell(v(x,t)) q(u(x,t)), \ x \in \Omega, \ t > 0, \end{cases} \tag{3}$$

and

$$\begin{cases} \dfrac{du(x,t)}{dt} = B(u(x,t)) - f(u(x,t)) v(x,t), \ x \in \Omega, \ t > 0, \\ \dfrac{dv(x,t)}{dt} = d \Delta v(x,t) + [\beta f(u(x,t)) - \mu] v(x,t), \ x \in \Omega, \ t > 0, \end{cases} \tag{4}$$

respectively. Under the assumptions that $p(u)$, $g(v)$, $h(u)$, $\ell(v)$ and $q(u)$ are smooth functions satisfying some monotonic conditions, the authors [38] established the existence of traveling wave solutions for systems (3), and Huang [39] still showed the existence of traveling wave solutions for systems (4) without monotonic assumptions of the functions $f(u)$ and $B(u)/f(u)$. Just recently, On the basis of Huang [39], Li and Xiao [14] proceeded to address the problem of the existence of traveling wave solutions for the following diffusive predator–prey type systems with nonlinear density-dependence

$$\begin{cases} \dfrac{du(x,t)}{dt} = B(u(x,t)) - f(u(x,t)) v(x,t), \ x \in \mathbf{R}, \ t > 0, \\ \dfrac{dv(x,t)}{dt} = d \Delta v(x,t) + \beta v(x,t) f(u(x,t)) - h(v(x,t)), \ x \in \mathbf{R}, \ t > 0, \end{cases} \tag{5}$$

and concluded that the traveling wave solutions established in Huang [39] can be preserved in the presence of the nonlinear density dependence for predator. The results obtained in [14,38] and [39] are so general that they generalize almost all relevant results in the references mentioned above, for example, they can be also applied to the diffusive Ivlev type predator–prey systems, which seems not to have been discussed before.

Let us summarize briefly the employed methods of getting a traveling wave. The methods of phase-plane analysis [25] and the method involves the study of an integral equation [26], are suitable for the second-order scalar reaction–convect ion–diffusion equation. For more higher order scalar equation, such as the extended Fisher–Kolmogorov equation, Peletier and Troy [27] introduced four methods, i.e., topological shooting based on a elaborate analysis, Hamiltonian methods, Variational methods based on a functional and methods based on the Maximum Principle. As mentioned above, Dunbar [30–32] started to investigate traveling wave of a diffusive predator–prey system using the method of topological shooting based on the Wazewski's topological principle and LaSalle's invariance principle. Though many authors studied the traveling wave using another methods of the index theory [33,34], the fixed theory [40], and Fenichel's geometric singular perturbation theory [24], Dunbar's method is the predominant one and was employed by so many mathematicians, see references