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A two-grid decoupled algorithm for fracture models

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ABSTRACT

In this paper, we consider a two-grid decoupled algorithm for two categories of fracture models. In the mixed Darcy/Darcy fracture model, fluid flow in fractures as well as in the surrounding medium is governed by Darcy's law. And in the mixed Darcy–Forchheimer/Darcy fracture model, flow in fractures is governed by Darcy–Forchheimer's law while that in surrounding matrix is governed by Darcy's law. In the proposed two-grid method, we use a coarse grid approximation to the interface coupling conditions for decoupling mixed problems in fractures and surrounding matrix. Error estimates show that the two-grid decoupled algorithm retains the same order of approximation accuracy as the coupled algorithm for the pressure p on H^1 semi-norm and the velocity $\mathbf{u}$ on $(L^2)^2$ norm. Numerical experiments are carried out to verify the accuracy and efficiency of the decoupled algorithm, in which the computation times are reduced greatly compared to the coupled method.

1. Introduction

In this paper, we consider a single phase fluid flow in porous media with fractures, which has aroused increasing interest of researchers [1–5]. Modeling flow in porous media is difficult and the corresponding mathematical problem is complicated. In particular, fractures are taken into account. The permeabilities of fractures are much higher or lower (due to crystalization) than that of surrounding matrix, thus fractures play important roles on flow in the media acting as privileged channels or barriers. In addition, there are interactions between fractures and surrounding domains. Compared to the size of the whole domain, the fracture generally has at least one dimension with a very small width, and an idea treating fractures as $(n-1)$-dimensional interfaces in the n-dimensional domain was proposed in [1] for high permeability fracture models. In [2], a model was presented to generalize the earlier models and it was improved to handle both large and small permeability of fractures. For models with more permeable fractures, fluid tends to flow into fractures and along them. Fractures can be seen as fast pathways in this case. The normal component of the velocity should not be expected to be continuous across fractures. While for models with less permeable fractures, it is easy to see that the fluid has a tendency to avoid fractures. Fractures act as geological barriers. The pressure will not be identical on both sides of fractures. Hence, nonstandard Robin type conditions on the interface were proposed in the fracture model of [2], which coupled a flow equation along the fracture with equations in surrounding domains. We also refer to [3–6] for similar models. For all of the above models, the linear Darcy's law is used as the constitutive law for flow in fractures as well as in the surrounding domains. However, for high-ranged velocity, Darcy's law cannot fit well with experiments, and a nonlinear correction term should be added, the Forchheimer term [7,8]. For fractures with large enough permeability, Darcy's law will be replaced by Darcy–Forchheimer's law. A model coupling Darcy–Forchheimer flow in the fracture and Darcy flow in the rest of the domain was carried out in [9], but the pressure was assumed to be continuous in this model. In [10] (2013), a similar model without the continuity

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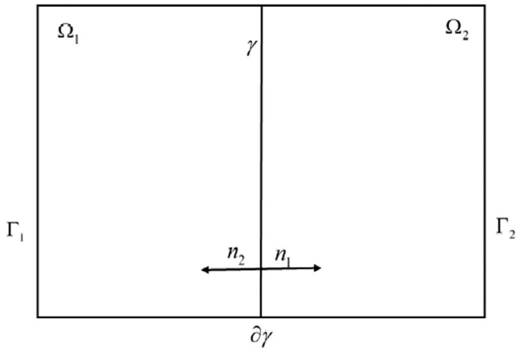

Fig. 1. The domain of the fracture model.

of pressure was derived, and the existence and uniqueness of the mixed finite element approximation were analyzed. There are also some researches on numerical methods of fracture models, such as an extended finite element method in [11], the mixed finite element with domain decomposition method in [12], the mixed finite element method with nonconforming grid in [5], and a block-centered finite difference method in [13]. In our paper, a decoupled two-grid algorithm is proposed to decouple the aforementioned fracture models in [2] and [10], using the coarse grid approximations on the interface. The decoupled local problems can be solved individually, and even on parallel processors, which is efficient and convenient.

The fracture model is one of multi-modeling problems consisting of different equations in fractures and surrounding domains, which are coupled via interface conditions. The aforementioned numerical approaches solve coupled fracture models directly. However, different equations defined in different regions are varied in type, such as coupling linear and nonlinear systems, and interface conditions include variables in different domains, which results in very complex algebraic structures. Another approach is to decouple mixed models firstly, and then local problems are solved by local solvers. There are some decoupling methods based on domain decomposition, for example, Quarteroni and Valli [14], Lagrange multiplier techniques [15], which have been widely used for multi-modeling problems. In [16] and [17], Xu proposed a new two-grid discretization technique for solving nonsymmetric and indefinite partial differential equations. Then in [18] and [19], the two-grid method was successfully applied to decouple multimodel problems, the mixed Stokes/Darcy model and the mixed Navier–Stokes/Darcy model. As shown in [18], the basic idea of the two-grid algorithm is to first solve a coupled problem on a much coarser grid, and then we get rough approximations of variables defined on the interface, which can be used to decouple the mixed model on the fine grid. So this procedure consists of a coupled solver on the coarse grid and a decoupled local solver on the fine grid. More details about the two-grid method can be seen in [20–22].

In this paper, we will apply the two-grid decoupled algorithm to the mixed Darcy/Darcy model in [2] and the mixed Darcy–Forchheimer/Darcy model in [10]. Traditional finite element methods for fracture models result in coupled and even nonlinear discrete problems. Computational cost and numerical difficulty increase largely as the mesh size decreases. There are many advantages for us to use the decoupled algorithm for mixed models. Firstly, we just need solve the coupled problem on a coarse grid, and on the fine grid, we solve local problems in subdomains individually, so numerical difficulty is reduced in comparison with the coupled method. Secondly, optimized local solvers can be applied to local problems. Especially for the Darcy–Forchheimer/Darcy fracture model, since the equation of the fracture is nonlinear, traditional methods will result in a coupled nonlinear discrete problem. But if the decoupled method is used, we only solve nonlinear system in the fracture instead of in the whole domain. Finally, local problems on the fine grid can be solved in a parallel multiprocess, and computational cost will be further reduced.

The rest of this paper is organized as follows. In Section 2, the fracture model coupling Darcy flow in both the fracture and surrounding matrix is described. And some notations are introduced. In Section 3, the two-grid decoupled algorithm using a coarse grid approximation on the interface is proposed for the Darcy/Darcy fracture model. In Section 4, error estimates for the proposed two-grid decoupled method are discussed, which shows that the two-grid decoupled method maintains the same order of convergence for pressure and velocity as the coupled method. In Section 5, we extend the two-grid decoupled algorithm to the mixed Darcy–Forchheimer/Darcy fracture model. In Section 6, numerical experiments for both coupled fracture models are carried out. Numerical results confirm our theoretical analysis, and computational time of decoupled method is reduced greatly compared to the coupled one. Finally, we give a brief conclusion of our work in Section 7.

2. The mixed Darcy/Darcy fracture model

We firstly consider the mixed Darcy/Darcy fracture model derived in [2], which couples 2-dimensional linear elliptic equations in surrounding domains with a 1-dimensional linear elliptic equation on the fracture. For simplicity, let $\Omega = [a_1, a_2] \times [b_1, b_2]$ be a rectangular domain in $\mathbb{R}^2$ with the boundary Γ and let $\gamma = \{x = x_f\} \times [b_1, b_2] \subset \Omega$ be a one-

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