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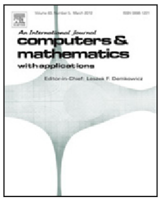

Dynamical survey of the dual power Zakharov–Kuznetsov–Burgers equation with external periodic perturbation

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A B S T R A C T

We investigate the dynamical behavior of the newly introduced dual power Zakharov–Kuznetsov–Burgers equation. Using bifurcation theory of planar dynamical systems, we study bifurcations of traveling wave solutions of the dual power Zakharov–Kuznetsov–Burgers equation in presence and absence of viscosity (μ) effect. In presence of an external periodic perturbation, we discuss the periodic and chaotic motions of the perturbed dual power Zakharov–Kuznetsov–Burgers equation by analyzing phase projection analysis, time series analysis, Poincaré section and sensitivity analysis. The effect of viscosity (μ) plays an important role in the transition from chaotic motion to periodic motion of the perturbed dual power Zakharov–Kuznetsov–Burgers equation.

1. Introduction

Nonlinear partial differential equations (NLPDEs) are widely used to describe physical phenomena in various aspects of physical areas, such as fluid dynamics, plasma dynamics, condensed matters, lattice dynamics, nonlinear optics etc. [1–6]. The investigation of nonlinear traveling wave solutions and integrability of different standard NLPDEs, such as the KdV equation [7], KP equation [8], ZK equation [9], sine-Gordon equation [10], KdV-Burgers equation [11], nonlinear Schrödinger equation [12] has tremendous application in nonlinear physical systems. In particular the completely integrable NLPDEs have extensive practical application from both mathematical and physical point of view [13–15]. It is to be mentioned that a completely integrable nonlinear evolution equation possesses some interesting properties such as Lax pair, N-soliton solution, infinite conservation laws, Painlevé property and bi-Hamiltonian structures. Nguetcho et al. [16] studied the bifurcations of phase portraits of a singular nonlinear equation of the second class and obtained various sufficient conditions leading to the existence of propagating wave solutions. Jiang et al. [17] investigated the CH-γ equation applying the bifurcation theory with phase portraits analysis and obtained bounded traveling wave solutions (solitary waves, periodic cusp waves and peakons). Douvagai et al. [18] investigated electromagnetic wave solitons (bright and dark solitons as well as kink and bell shape solitary wave solutions) in optical metamaterials. Saha [19] reported various sufficient conditions to guarantee the existence of solitary wave, periodic wave, periodic cusp wave and compacton of the generalized KP-MEW equations employing the bifurcation theory of planar dynamical systems. With the help of bifurcation theory of dynamical systems, Liu and Yan [20] presented the bifurcation of solitary wave, periodic wave, kink wave, anti-kink wave and breaking

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wave solutions. However, in several practical physical models there exist various periodic perturbations whose nature may be different depending on the different aspect of the physical process. Presently the study of nonlinear evolution equations in presence of external periodic perturbations [21,22] has become one of the intense research activity. It is interesting to observe that a completely integrable nonlinear model equation cannot describe chaotic phenomenon, but the influence of a periodic perturbation to an integrable nonlinear model equation may lead to chaotic dynamics, such as perturbed sine-Gordon equation [23], perturbed KdV-Burgers equation [24] and perturbed Schrödinger equation [25]. The investigation of chaotic features of a nonlinear physical system in presence of an external periodic perturbation can be executed through different possible routes: (i) period doubling route to chaos, (ii) quasiperiodic route to chaos and (iii) intermittent bifurcation and crisis.

The generalized two-dimensional Zakharov–Kuznetsov equation containing dual-power nonlinearity [26–28] is

$$q_t + (aq^n + bq^{2n})q_x + c(q_{xx} + q_{yy})_x = 0\,, \tag{1}$$

where n and $2n$ denote the dual-power laws ($n > 0$) and $a,\ b$ and $c \neq 0$ are real numbers. With the help of numerical computation and auxiliary ordinary differential equations, Zhang [29] applied a new generalized algebraic method to the generalized Zakharov–Kuznetsov equation (1) containing higher-order nonlinear terms to obtain a series of new and more general traveling wave solutions. Biswas et al. [28] reported the 1-soliton solution of Eq. (1) in the form $q(x, y, t) = A/[D + \cosh(B_1x + B_2y - vt)]^{1/n}$ with higher-order nonlinearity by employing the solitary wave ansatz method where A and D are determined in terms of other parameters. Seadawy [30] applied the reductive perturbation method to obtain solitary traveling waves of three-dimensional modified ZK equation and also investigated the stability of solitary traveling waves.

If $n = 1$, Eq. (1) transforms to the dual power ZK equation given by

$$q_t + (aq + bq^2)q_x + c(q_{xx} + q_{yy})_x = 0\,. \tag{2}$$

Yan et al. [31] considered two types of symmetry of the generalized ZK equation (2) via a direct symmetry method to obtain some symmetry reductions and new explicit solutions of Eq. (2).

The generalized Burgers equation is

$$q_t + aqq_x = bq_{xx}\,, \tag{3}$$

where x is a space variable, t is time, q is plasma potential, a is the nonlinear coefficient and $b = \frac{\eta}{2}$ is the coefficient of dissipation and η is known as the coefficient of viscosity. The evolution equation (3) governs nonlinear shock waves in dissipative plasmas [32,33].

Zhen et al. [34] investigated the Zakharov–Kuznetsov–Burgers (ZKB) equations for the dust ion- acoustic waves in dusty plasmas and obtained shock-like and soliton solutions. They also considered external perturbations in dusty plasmas and studied the perturbed ZKB equation via some qualitative and quantitative methods. In [35], they have studied dynamical behavior of the quantum Zakharov–Kuznetsov equations in dense quantum magnetoplasmas in which external periodic force yields chaotic motions. Yang et al. [36] used a multiscale and perturbation method and obtained a ZK-Burgers equation which describes three dimensional Rossby waves. Furthermore they obtained conservation laws of the Rossby waves, rational solutions and discuss the dissipation and chirp effect of three-dimensional Rossby solitary waves. A similar result is also obtained for ZK-BO equation and ZK-mZK equation by Yang et al. in [37] and Zhao et al. in [38]. In [39], a new ZK-ILW equation is derived by using multiscale analysis and perturbation method in finite depth stratified atmosphere and the analytical solutions and conservation laws are derived. Also the fission process of algebraic gravity solitary waves is studied. In [40], the time fractional generalized Boussinesq equation for Rossby solitary waves with dissipation effect in stratified fluid is derived using reduction perturbation method and the exact solutions and conservation laws are discussed briefly.

In this present work, we introduce the ZK-Burgers equation with dual power nonlinearity for the first time as

$$q_t + (aq + bq^2)q_x + c(q_{xx} + q_{yy})_x + \mu(q_{xx} + q_{yy}) = 0\,, \tag{4}$$

where a, b are coefficients of nonlinear terms, c is the coefficient of dispersion term and μ is the coefficient of dissipation. Eq. (4) describes nonlinear traveling waves in dissipative plasmas and fluid systems [34]. We generalize Eq. (4) by introducing an external periodic perturbation f to the dual power ZK-Burgers equation and introduce the perturbed dual power ZK-Burgers equation as

$$q_t + (aq + bq^2)q_x + c(q_{xx} + q_{yy})_x + \mu(q_{xx} + q_{yy}) = f_x\,. \tag{5}$$

We study the dynamical behavior of the traveling wave solutions of the perturbed dual power ZK-Burgers equation (5).

Consider the traveling wave transformation $\xi = lx+my-vt$, where l and m are direction cosines of the line of propagation of the traveling wave with velocity v in the xy-plane such that $l^2+m^2 = 1$ to discuss all possible periodic and chaotic motions of Eq.(5). Then Eq.(5) reduces to

$$-vq_\xi + l(aq + bq^2)q_\xi + lcq_{\xi\xi\xi} + \mu q_{\xi\xi} = lf_\xi\,. \tag{6}$$

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