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Modeling origin-destination uncertainty using network sensor and survey data and new approaches to robust control

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ABSTRACT

This study develops new methods for network assessment and control by taking explicit account of demand variability and uncertainty using partial sensor and survey data while imposing equilibrium conditions during the data collection phase. The methods consist of rules for generating possible origin–destination (OD) matrices and the calculation of average and quantile network costs. The assessment methodology leads to improved decision-making in transport planning and operations and is used to develop management and control strategies that result in more robust network performance. Specific contributions in this work consist of: (a) Characterization of OD demand variability, specifically with or without equilibrium assumptions during data collection; (b) exhibiting the highly disconnected nature of OD space demonstrating that many current approaches to the problem of optimal control may be computationally intractable; (c) development of feasible Monte Carlo procedures for the generation of possible OD matrices used in an assessment of network performance; and (d) calculation of robust network controls, with state-of-the-art cost estimation, for the following strategies: Bayes, p-quantile and NBNQ (near-Bayes near-Quantile). All strategies involve the simultaneous calculation of controls and equilibrium conditions. A numerical example for a moderate sized network is presented where it is shown that robust controls can provide approx. 20% cost reduction.

1. Introduction

State of the art in transport network modeling is based on the estimation of travel demands between origins and destinations in the form of a trip table by various techniques. Once a trip table has been estimated by any of these methods, analysts typically proceed to evaluate the performance of various planning, design or operational alternatives for the transportation network assuming the demands established in the table are unaffected by those alternatives. The analysis culminates with a network assignment procedure that generates the volumes and costs associated with those volumes on the paths and links of the network. This classical approach is realized by network equilibrium models and algorithms as described in detail by Patriksson (1994) and Florian and Hearn (1995), and more recently by Marcotte and Patriksson (2007). Since the input data are, generally, noisy or incomplete the resulting demand data used in the analysis involve considerable uncertainty. Furthermore, the assumption of immutability of the demands in the face of changing network conditions is quite unrealistic.

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This study develops robust optimization (RO) methods for network assessment and control that take explicit account of demand variability and uncertainty while observing equilibrium conditions on the observation space. The methods consist of rules for generating possible origin-destination (OD) matrices and the calculation of quantile network costs. This approach can lead to improved decision-making in transport planning and operations and can be used to develop management and control strategies that result in more robust network performance. As pointed out by Bertsimas et al. (2011), RO is an approach to optimization under uncertainty, in which the uncertainty model is not stochastic, but rather deterministic and set-based. Instead of seeking to immunize the solution in some probabilistic sense to stochastic uncertainty, this approach seeks a solution that is feasible for *any* realization of the uncertainty in a given set. Whereas, in general, a robust solution is not optimal for all realizations of the uncertain data, this approach performs well even under severe case scenarios. A number of strategies that are developed in this paper are a manifestation of this approach. The motivation for this approach is twofold. First, the model of set-based uncertainty is a suitable application of parameter uncertainty. Second, computational tractability is an important motivation and goal. These characteristics are responsible for the considerable success of RO in many application areas.

Several approaches exist for network analysis with OD uncertainty. For example, Xie et al. (2010, 2011), derive a maximum entropy estimate of the OD matrix using available link flow measurements. For this estimate, equilibrium assignments and total costs can be determined as a function of the signal controls and then minimized. Maximum likelihood and generalized least squares methods have been used by other authors (van Zuylen and Willumsen, 1980; Bell, 1991; Nie et al., 2005). Numerous other approaches have been proposed in the literature, see Bell and Iida (1997), Yang et al. (1994), and Yang (1995). Bierlaire (2002) proposes a measure of quality for estimated OD tables, called the *total demand scale*. It measures the uncertainty due to the network topology and the route choice assumptions and is complementary with other measures of quality. However, in many cases, when existence of a compatible equilibrium is imposed as a constraint on the uncertainty space and some criterion is optimized to choose a single OD matrix for analysis, the problem may become (as shown below) infeasible.

In Jones et al. (2013), the unknown OD matrices were characterized by the space of all non-negative matrices with fixed row and column sums, corresponding to the given total out- and in-flows in vehicles per hour as specified by sensors at sources and sinks. Probability distributions on this space were then proposed based on additional data to represent the relative uncertainty of actual OD values, and Bayesian and minimax strategies were developed for calculating optimal traffic signal controls. For instance, the Bayes optimal controls are signal settings which minimize the expected system cost at equilibrium flows, or system-optimal flows, with respect to the probability distribution.[1]

Other authors have applied RO techniques to address uncertainty in special cases of traffic control. Yin (2008) investigates methods of signal optimization for pre-timed signal control under demand fluctuations. For the case of an isolated intersection, he develops a timing plan whose performance is near optimal in an average sense, and is fairly stable under any realization of uncertain traffic flows. Ukkusuri et al. (2010) present a dynamic system-optimal signal control model with a stochastic representation of the OD demands within an RO framework. Three probability distributions are considered for each OD demand from which a set of discrete scenarios is generated. Traffic flow and performance is represented by means of an embedded cell transmission model. This approach is applied in a fairly simple network with a single multi-phase signalized intersection offering limited route choices.

The interdependency between signal timings and user route choices has been described extensively in the literature. Comprehensive discussions can be found in Lee and Machemehl (2005) and in Mitsakis et al. (2011). Simonelli et al. (2012) consider the optimal network sensor location problem accounting for the variability of the OD matrix estimate. Castillo et al. (2015) provide an integrated approach, based on mathematical optimization, for the sensor location, flow observability, estimation and prediction problems in traffic networks.

The presentation in this paper proceeds as follows: Section 2 initially defines the uncertainty space by characterizing the space of OD matrices (termed *feasible*) in a network which are consistent with observations expressed as a set of path flow constraints. Assuming there is an equilibrium assignment that is also consistent with the observations, the uncertainty space is further narrowed to a space of *admissible* OD matrices. Section 3 provides a demonstration that the latter, more restricted OD space, may be highly disconnected. This has significant implications on the development of algorithms for calculating robust controls. Section 4 presents Monte Carlo methods for generating OD matrices that (a) satisfy observed constraints, and (b) also provide equilibrium solutions satisfying the constraints. Section 5 applies the methodology in a moderately sized network and calculates robust controls for two uncertainty sets: a Monte Carlo sample size of 100 feasible matrices consistent with the measured data and an independently generated Monte Carlo sample size of 100 admissible matrices having equilibrium flows that are consistent with the data. Controls and associated network costs are calculated for the following strategies: *Bayes, p-quantile and NBNQ* (*near-Bayes near-Quantile*). All calculations are based on the state-of-the-art cost function given in HCM (2010). Furthermore, Appendix A provides a proof that the total cost function using the HCM (2010) formula is separately, but not jointly, convex in the controls and in the flows. This function serves as a basis for all the subsequent cost calculations.

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[1] Jones et al. (2013) labeled the calculation *ECO* (for equilibrium constrained optimization); i.e., calculating signal controls that minimize network travel costs while observing the equilibrium constraints.

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