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Relaxing the high-frequency gain sign assumption in direct model reference adaptive control[☆]

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ABSTRACT

A new, high performance, solution to the classical problem of direct model reference adaptive control for linear time-invariant systems with *unknown sign of the high frequency gain* is reported in the paper. The proposed algorithm directly estimates this parameter with the only required prior knowledge of a lower bound on its absolute value. To avoid the possible appearance of singularities in the controller calculation a switched projection mechanism is introduced to change, if needed, the sign of the estimate. The recently introduced *dynamic regressor extension and mixing* estimator is used to ensure monotonicity of the estimation error of the high frequency gain, guaranteeing that the switching appears (at most) once and avoiding the possible appearance of chattering—that may happen in classical gradient-based algorithms. Comparative simulations with the Nussbaum gain-based and gradient estimators illustrate the dramatic *performance improvement* of the proposed controller.

1. Introduction

Model reference adaptive control (MRAC) is unquestionably the most widely studied problem in the adaptive literature that has a very long history going back to the 1950's and extending to the present time. The first attempts to solve the MRAC problem followed the classical path of designing an observer, that had to be made adaptive because of the unknown plant parameters, and then feeding back the observed state (see [6]). Very little success was, however, obtained pursuing these lines—essentially because of the difficulty of simultaneously estimating state and parameters. A major breakthrough, essentially due to [2,8], was the introduction of the so-called *direct* control parameterization (see Lemma 1 below), which revealed that the estimation of the plant state could be obviated and only a "good" estimation of the controller parameters was needed to achieve the asymptotic reference model output tracking objective. The intrinsic simplicity of this parametrization motivated the overwhelming majority of the researchers to pursue this line of reasoning and concentrated their efforts into the development of suitable parameter estimators. The interested reader is referred to [12] for a vivid description of the history of MRAC as well as to the existing textbooks [5,14,19] for further information on it.

As it is well-known, the direct control parameterization, refered to as *output-error* parameterization[1] in [19], leads to a *bilinear* regression form, where the parameter that corresponds to the high frequency gain—denoted k_p in the sequel—appears multiplying the controller parameters. This difficulty can be overcome assuming the knowledge of the sign(k_p), under which a globally convergent output-error MRAC may be designed introducing an *overparameterisation* of the regressor and a *normalisation* in, now classical, augmented error-based estimators [5,14,19]. It was shown that these algorithms enjoy the fundamental "self-tuning property", that is, that global tracking is ensured for all reference signals—without imposing the stronger parameter convergence requirement. The use of normalisation and overparameterisation, however, comes with a very high tag for the overall performance of the scheme. Indeed, as thoroughly discussed in [10,16,19], overparameterisation hampers parameter convergence while normalisation "slows down" the adaptation and severely penalizes the parameter convergence rate. As shown in [16], this below par performance can be partially overcome using (the unnormalised) Morse's high order tuners [11], but the additional information of an upper

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[1] The term "output-error" has been used in [7] to refer to a completely different construction used in identification and adaptive control.

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bound on k_p is required and the scheme is significantly more involved.

A major theoretical breakthrough for this problem is due to Nussbaum [15] who, motivated by a conjecture in [9], showed that the sign of k_p is not necessary for stabilization in MRAC. Nussbaum's solution relies on the introduction of a function that changes periodically the sign of the estimator vector field in a "gain scheduling-like" fashion. It is clear that this kind of algorithms is only of theoretical interest since their transient performance is intrinsically bad and practically inadmissible—as it has been repeatedly reported in the literature.

Schemes that require the division by the estimate of k_p in the controller calculation, *e.g.*, the one proposed in Section 4.5.2 of [5] and the other one presented in the paper [4], most incorporate a switched projection to avoid singularities. There are two drawbacks to this approach, on one hand, to the best of the authors' knowledge, no proof of global tracking for this scheme has been reported in the literature without an *unverifiable* assumption of persistency of excitation (PE) of the regressor. On the other hand, there is no guarantee that the switching happens only a finite number of times nor the possible appearance of *chattering* phenomena.

In this paper, a new solution to the problem with improved transient performance is reported, which includes the following modifications:

(M1) Abandon the bilinear model mentioned above, and adopt instead the overparameterized *linear regression*.
(M2) Introduce a new factorization of the parameter *estimates* to update directly the controller parameters.
(M3) Instead of classical gradient estimators we use the recently introduced *dynamic regressor extension and mixing* (DREM) estimator from [1].

The use of a linear parameterization is essential to apply the DREM estimator. Unfortunately, the estimation law still involves the division by an estimate of k_p. Therefore, similarly to the classical schemes, a switched projection of this estimate is added to keep it away from an a priori known band around the zero value. To avoid the undesirable chattering phenomena indicated above we exploit a key feature of DREM: that it ensures *monotonicity* of the estimation error of the parameter k_p, ensuring that the switching appears (at most) once. The monotonicity property holds for all reference signal. However, global tracking can only be ensured for reference signals that satisfy an excitation requirement, which holds true if the aforementioned PE assumption on the regressor of classical schemes is satisfied.

The remainder of the paper is organized as follows. Section 2 formulates the MRAC problem addressed in the paper and briefly reviews the current literature available on this topic. An MRAC, with a gradient-based procedure to estimate the controller parameters using the new factorization mentioned above, is given in Section 3. Section 4 contains our main result, namely, the description of the DREM estimator and its stability properties when applied in a MRAC scheme. Comparative simulations with the classical Nussbaum gain-based and gradient estimators, which illustrate the significant *performance improvement* of the proposed controller, are presented in Section 5. The paper is wrapped-up with concluding remarks in Section 6.

2. The MRAC problem with unknown sign(k_p)

2.1. Problem formulation

We are interested in the classical problem of relaxing the knowledge of the high frequency gain in MRAC of the scalar linear time-invariant (LTI) continuous-time plant

$$D(p)y = k_p N(p)u, \tag{1}$$

where y, u are the plant output and input, respectively, $D(p)$ and $N(p)$ are monic and coprime polynomials of degree n and m, respectively, $p := \frac{d}{dt}$, $\rho := n - m \geq 1$ and $k_p \in \mathbb{R}$ is the high frequency gain. The parameters of $D(p)$ and $N(p)$ are unknown.

We make the following assumptions regarding the plant.

(A.1) $N(p)$ is a Hurwitz polynomial.
(A.2) n and ρ are known.
(A.3) A constant $\underline{k}_p \in \mathbb{R}_+$ verifying

$$|k_p| \geq \underline{k}_p \tag{2}$$

is known.

The MRAC objective is to asymptotically drive to zero the tracking error

$$e = y - \frac{k_m}{D_m(p)} r \tag{3}$$

where $D_m(p)$ is a monic, Hurwitz polynomial of degree ρ, $k_m \in \mathbb{R}$ and r is a bounded reference.

2.2. Remarks on the assumptions

[R1] Assumptions A.1 and A.2, though somehow restrictive, are standard in MRAC (see, for example, [5,14,19]).

[R2] Conspicuous by its absence is the assumption of knowledge of the *sign of the high–frequency gain* of the plant k_p.[2] Relaxing this assumption is the main subject of interest in this note. Instead of its sign we assume that k_p is bounded away from zero—by a known value $\underline{k}_p$—as indicated in (2). Although this is a sensible assumption in all practical scenarios, it will be shown below that the transient behavior is degraded if $\underline{k}_p$ is too small.

[R3] We have assumed $\rho \geq 1$ to simplify the notation in the sequel. As will become clear later, the scheme proposed here—with the adequate technical changes—applies as well to the case $\rho = 0$.

[R4] Without loss of generality we have selected the reference model without zeros, a scenario usually adopted in MRAC designs. The theory can be extended *verbatim* for general reference model transfer functions.

2.3. A key lemma

Instrumental for the development of MRAC is the lemma below, known as the direct control model reference parameterization, first established by [2,8] (see also [5,19] for a modern derivation of the result).

Lemma 1. *Consider the plant (1) and the tracking error (3). There exists a vector $\theta \in \mathbb{R}^{2n}$ such that*

$$e = \frac{k_p}{D_m(p)}(u - \theta^\top \phi) + \epsilon_t, \tag{4}$$

where $\phi \in \mathbb{R}^{2n}$ is the regressor vector given by

$$\phi = \frac{1}{\lambda(p)} \mathrm{col}(u, \dot{u}, \ldots, u^{(n-2)}, y, \dot{y}, \ldots, y^{(n-2)}, \lambda(p)y, \lambda(p)r) \tag{5}$$

with a designer-chosen monic, Hurwitz polynomial $\lambda(p)$ and ϵ_t is an exponentially decaying term due to initial conditions.[3]

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[2] In time-domain this is tantamount to the knowledge of the sign of the instantaneous step-response.

[3] These terms will be omitted (without loss of generality) in the sequel.

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