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A novel semi-analytical method based on equivalent energy principle to obtain J resistance curves of ductile materials

Yun-qiang Peng, Li-xun Cai*, Hui Chen*, Chen Bao, Yi He

Applied Mechanics and Structure Safety Key Laboratory of Sichuan Province, School of Mechanics and Engineering, Southwest Jiao Tong University, Chengdu 610031, China

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ABSTRACT

Four kinds of configurations represented by compact tension (CT) specimen, single edge-notched bending (SEB) specimen, single edge-notched tension (SET) specimen and C-shaped inside edge-notched tension (CIET) specimen were simulated by ANSYS 14.5 under plane strain conditions and three dimension (3D) conditions for determining the parameters of load-displacement, J integral-load etc. semi-analytical expressions. And a novel semi-analytical method is proposed to obtain J resistance curves of ductile materials on the basis of equivalent energy principle. In order to examine the validity of semi-analytical expressions, the fundamental curves (e.g., load-displacement, J integral-load curves etc.) predicted by the expressions were compared with the curves obtained from finite element analysis (FEA). It can be observed that they agree well with each other. Moreover, the real-time crack length and J integral of growing cracked specimen at arbitrary loading points can be determined through solving the explicit expressions. And a corrected formula of J integral was obtained in order to consider the influence of crack growth of sharp cracked specimen. Further, the J resistance curves of Cr2Ni2MoV for CT specimens and 26NiCrMoV11-5 for CIET specimens were successfully obtained via the new method, which show good agreement with the results determined by traditional normalization method. And the crack lengths predicted by the method match with those obtained by physical measurement on the fracture surface of specimens. Meanwhile, the critical J_C which represents the sharp cracked specimen beginning to growth can also be easily determined.

1. Introduction

The fracture toughness measurement of ductile material plays a key role in safety assessment of structures, the key point of which is to measure the J integral of crack components. The theoretical concept of J integral was developed in 1967 by Cherepanov [1] and in 1968 by Rice [2] independently, and it was latterly proved by Hutchinson [3], Rice and Rosengren [4] that the J integral can represent the intensity of stress-strain field around the crack tip of crack components for elasto-plastic materials, named HRR field. In 1972, Begley and Landes [5] proved that J integral can be taken as a control parameter to characterize the quasi-static fracture behaviors of ductile materials. Further, Shih and Hutchinson [6,7] obtained the numerical fully-plastic expressions of load-plastic displacement (P-h_p) relation and plastic J integral-load (J_p-P) relation etc. for different configurations based on refined finite element analysis (FEA) [8-10] in 1976, and an engineering approach for elastic-plastic fracture analysis was proposed at the same time. Based on the research findings of Shih and Hutchinson [6,7], Kumar et al. [11] wrote an elastic-plastic fracture mechanics manual, named EPRI manual. The manual gave a feasibility for elasto-plastic safety assessment of crack components, but all the elastic-plastic equations provided in the manual are implicit, in order to obtain the J resistance curves of materials, a mass of computations are required in order to obtain the crack driving force diagram via the tables given in the manual. Furthermore, the EPRI method didn't consider the influence of crack growth of sharp cracked specimen. Analogous, Qian et al. [12,13] proposed a hybrid method to obtain the J resistance curves of ductile materials by combining experiments and finite element analyses, but it has the similar drawbacks with EPRI method. Until now, it's still a difficult process to obtain the J resistance curves of ductile materials theoretically, so the experimental methods such as unloading compliance method (UCM) [14,15], normalization method (NM) [16–18], and separable S_{pb} method (SSM) [19,20] are widely used. However, the accurate measurement of real-time crack length of growing cracked specimen by these experimental methods is still difficult. For examples, the unloading compliance is sensitive to the crack length of growing cracked specimen in the UCM, which usually causes more deviations in predicting the real-time crack length; the NM requires to measure the initial crack length and final crack length of growing cracked specimen, and the form of deformation function is determined empirically in order to obtain the real-time crack length of specimen; in SSM, the choice of reference blunt

* Corresponding authors.
E-mail addresses: lix_cai@263.net (L.-x. Cai), chen_hui5352@163.com (H. Chen).

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cracked specimen severely effect on the prediction of real-time crack length of growing cracked specimen. Additionally, the aforementioned experimental methods must determine the plastic factor η_p first in order to obtain the J integral of specimen. According to the research findings of Ernst and Paris [21], η_p is inexistent when the assumption of load separation principle cannot be met. Generally speaking, two methods represented by plastic work method [22] and load separation method [23] are usually used to obtain the plastic factor η_p, and the η_p is considered as a function only related to the crack length of specimen. However, Donato and Ruggieri [24] and Huang et al. [25] analyzed the value of η_p for SEB specimens by FEA, the results indicated that η_p is related to not only crack length but also material properties and loading force. Similar conclusions were obtained by Cravero and Ruggieri [26], Ruggieri [27] and Huang and Zhou [28] based on their research findings of η_p for SET specimens. Consequently, the J resistance curves of ductile materials obtained by traditional experimental methods are approximate results.

In this paper, several elasto-plastic semi-analytical expressions of load-displacement, energy-load and J integral-load relations are obtained on the basis of Chen–Cai equivalent energy principle [29–31]. The validity checks of these formulas were performed by FEA. Finally, a novel method is applied to obtain the J resistance curves of ductile materials by compact tension (CT) specimen and C-shaped inside edge-notched tension (CIET) specimen [32].

2. The theoretical model

Ramberg–Osgood constitutive relation is the most common model to describe power-law hardening materials in fracture mechanics. It has a form as

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^N \tag{1}$$

where σ represents true stress and ε true strain of material, respectively. ε_e is elastic strain and ε_p plastic strain. E is elastic modulus, K strength coefficient and N hardening exponent.

When only considering the fully plastic part of Eq. (1), the total plastic strain energy U_p of arbitrary deformation body can be given as

$$U_p = \iiint_{\Omega} u_{p-\mathrm{eq}}(x, y, z)\,dx\,dy\,dz = u_{p-\mathrm{eqm}} V_p \tag{2}$$

where Ω is effective deformed zone of deformation body, $u_{p\text{-eq}}$ is equivalent plastic strain energy density of representative volume element (RVE) at arbitrary point (*x, y, z*) in the deformed zone, V_p is effective deformed volume of Ω, $u_{p\text{-eqm}}$ is RVE's plastic strain energy density at energy center ξ (x_ξ, y_ξ, z_ξ), and it can be written as

$$u_{p-\mathrm{eqm}} = \int_0^{\varepsilon_{p-\mathrm{eqm}}(h_p)} \sigma\, d\varepsilon_p = \frac{NK}{N+1}\varepsilon_{p-\mathrm{eqm}}^{1+1/N} \tag{3}$$

where h_p represents fully plastic displacement. Further, the total plastic strain energy U_p is expressed as [30]

$$U_p = \frac{NK}{N+1} V_P \varepsilon_{p-\mathrm{eqm}}^{1/N+1} = \frac{NKV^*}{N+1}\frac{V_P}{V^*}\varepsilon_{p-\mathrm{eqm}}^{1/N+1} \tag{4}$$

in which V^* is characteristic volume and $V^* = A^* h^*$, A^* is characteristic area and $A^* = WB(1\text{-}a/W)^m$ for crack components, W and B are width and thickness of crack specimen respectively, h^* is characteristic displacement and $h^* = W$, m is effective volume reduction coefficient related to crack length a of specimen, the aim of which is to make all the $P/(KA^*)$-h_p/h^*curves with different crack lengths normalized. P is loading force of specimen, which can be given by differentiating Eq. (4)

$$P = \frac{\partial U_p}{\partial h_p} \tag{5}$$

It's assumed that plastic effective volume and plastic equivalent strain can be given as

$$\begin{cases} \dfrac{V_p}{V^*} = k_1\left(\dfrac{h_p}{h^*}\right)^{k_2} \\ \varepsilon_{p-\mathrm{eqm}} = k_3\left(\frac{h_p}{h^*}\right)^{k_4} \end{cases} \tag{6}$$

where k_1 and k_2 are plastic effective volume coefficient and plastic effective volume exponent respectively, k_3 is equivalent plastic strain coefficient and k_4 equivalent plastic strain exponent.

Combining Eqs. (4)–(6), we have

$$\begin{cases} \frac{U_p}{U_p^*} = \left(\frac{h_p}{h^*}\right)^{m_p+1} \\ \frac{U_p}{U_p^*} = \left(\frac{P}{P_p^*}\right)^{1/m_p+1} \\ \frac{P}{P_p^*} = \left(\frac{h_p}{h^*}\right)^{m_p} \end{cases} \tag{7}$$

where $U_p^* = \frac{NKV^*}{N+1}k_1 k_3^{1+1/N}$, $m_p = \frac{k_4}{N} + k_4 + k_2 - 1$, $P_p^* = \frac{(1+m_p)NKA^*}{(N+1)}k_1 k_3^{1+1/N}$.

Analogously, the relations of P–h_e, U_e–P and U_e–h_e for linear elastic deformation of materials can be given as

$$\begin{cases} \frac{U_e}{U_e^*} = \left(\frac{h_e}{h^*}\right)^2 \\ \frac{U_e}{U_e^*} = \left(\frac{P}{P_e^*}\right)^2 \\ \frac{P}{P_e^*} = \left(\frac{h_e}{h^*}\right) \end{cases} \tag{8}$$

where $U_e^* = \frac{k_5 EV^*}{2}$, $P_e^* = k_5 EA^*$, U_e is elastic strain energy, h_e is elastic displacement.

For elastoplastic deformation of materials, the total energy U and total displacement h of crack components can be obtained by the sum of linear elastic and fully plastic contributions.

$$h(P) = h_e(P) + h_p(P) \tag{9}$$

$$U(P, h) = U_e(P, h_e) + U_p(P, h_p) \tag{10}$$

According to Eq. (9), the P–h relation of elastoplastic deformation of specimen can be written as

$$\left(\frac{P}{P_p^*}\right)^{\frac{1}{m_p}} + \frac{P}{P_e^*} = \frac{h}{h^*} \tag{11}$$

The dimensionless form of the above equation can be written as

$$(b_1\frac{P}{P^*})^{\frac{1}{m_p}} + b_2\frac{P}{P^*} = \frac{h}{h^*} \tag{12}$$

where

$$\begin{cases} P^* = \sqrt{P_p^* P_e^*} \\ b_1 = \left(\sqrt{\frac{P_e^*}{P_p^*}}\right) \\ b_2 = \sqrt{\frac{P_p^*}{P_e^*}} \end{cases} \tag{13}$$

Based on the energy definition of J integral, Eq. (14), proposed by Rice [33], the J integral of a specimen can be expressed as follows

$$J = J_e + J_P = -\frac{\partial U}{B\partial a} \tag{14}$$

$$J = \left(1 - \frac{a}{W}\right)^{m-1}\left[b_3\left(\frac{P}{P_p^*}\right)^{b_4} + b_5\left(\frac{P}{P_e^*}\right)^2\right] \tag{15}$$