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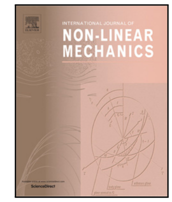

An accurate and efficient scheme for linear and nonlinear analyses based on a gradient-weighted technique

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ABSTRACT

An accurate and efficient gradient weighted finite element method (GW-FEM) is developed for linear elastic, free vibration and material nonlinear analyses. The new approach is based on the triangular and tetrahedral elements that can be generated automatically for any complicated geometries in 2D and 3D spaces. Shepard interpolation technique (SIT) is used to formulate the weighted gradient field considering the effect of the element itself and its adjacent elements sharing common edges (2D) or faces (3D). Due to the simple formulations, the SIT is easily implemented and coded in constructing the weighted gradient field. Both of the linear elastic and work-hardening-based elastic–plastic material models are incorporated in the GW-FEM for the linear and nonlinear analyses. The GW-FEM is then coupled with the total strain theory and projection method to solve the nonlinear elastic–plastic problem. Our numerical examples, including both of benchmark and practical engineering cases, reveal that GW-FEM provides superior performance in accuracy and efficiency, compared to the standard finite element method.

1. Introduction

Static, free vibration and elastic–plastic analyses of solid mechanics are commonly encountered by researchers in mechanical, civil and aerospace engineering. As the analytical solutions are only available for problems with very simple geometries, various numerical algorithms, including the finite element method (FEM) [1,2], the finite volume method (FVM) [3–5], the meshless finite element method (MFEM) [6–8], the boundary element method (BEM) [9–11] etc., have already been devised to simulate these behaviors in the past decades. Among these different methodologies, the finite element method is still so far the most popular and versatile tool.

The salient feature of FEM is that it enables continuums to be computed with ease by discretizing them into a finite number of elements. In engineering applications, researchers often resort to the quadrilateral and hexahedron elements that provide reliable results. However, the traditional quadrilateral and hexahedron elements have inherent shortcomings [12]: (1) the computation cost is enormous since the high-order interpolation and multiple integration points are required in each element, (2) the high demands for the shapes of element, such as positive Jacobian and ideal interior angles (all less than 180°), make it limited for practical application. Although the linear triangular and tetrahedral elements are simplistic and convenient for dealing with the problems with complex geometries, they often suffer from the well-known low accuracy deficiency. Recently, numerous algorithms have been introduced into the traditional FEM to enhance its performance, such as natural element method (NEM) [13], linear six-node "Triprism" element [14,15], central point-based discrete shear gap method (CP-DSG3) [16], partition of unity method (PUM) [17], and reproducing kernel approximation [18,19] and so on. These algorithms, providing local but continuous approximation functions, enjoy several advantages as compared to the standard FEM, especially for improving computational efficiency and accuracy. However, there are still several common problems have to be addressed urgently, *e.g.*, distorted meshes.

To overcome these drawbacks, a group of smoothed finite element methods (S-FEM) [20–24] were presented based on the gradient smoothing technique [25,26]. The node-based smoothed finite element methods (NS-FEM) [27,28] were developed in the frame of FEM. Feng et al. studied the static and dynamic analysis of Timoshenko beam [29] using the NS-FEM and developed a stable node-based smoothed finite element method (SNS-FEM) [27] to address the non-zero energy spurious modes [30] accompanying with NS-FEM. In order to eradicate the overly-soft behavior of NS-FEM, the edge-based smoothed finite

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element (ES-FEM) [30–34] is formulated for 2D analyses. It indicated the excellent performances in dealing with the solid mechanics [35], acoustic [36–38], crack growth [39,40], heat transfer [41,42] problems. The face-based smoothed finite element method (FS-FEM) [43] was also developed for the analyses of 3D problems, including geometrically non-linear solid mechanics [44] and visco-elastoplastic analyses [45]. However, the constructions of smoothed strain matrices using both ES-FEM and FS-FEM were laborious and cost a mass of physical memories.

Following the fundamental works mentioned above, a gradient weighted finite element method (GW-FEM) that combines the Shepard interpolation technique (SIT) and linear shape function is developed for the analyses of 2D and 3D solid mechanics. We use the three-node triangular or four-node tetrahedral elements that can be generated automatically for any complicated geometries to discretize the problem domain. For each independent element, a compacted supported domain is further formed based on the element itself and its adjacent elements sharing common edges/faces. Consider the advantages of a simple form in formulating the weighted function and of the resulting computing effectiveness, we use the SIT to construct the weighted gradient field. Then we compose the discretized system of equations based on the generalized Galerkin weak form. Our numerical examples, including both benchmark cases and practical engineering problems, demonstrate that the present method possesses superior performance compared to the standard FEM for linear and nonlinear problems, and is suitable for engineering application even with coarse mesh.

2. Basic equations for standard FEM

A 3D static elasticity problem is described using the following equilibrium equation in domain Ω bounded by Γ

$$\mathbf{L}^{\mathrm{T}}\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \quad \textit{in problem domain } \Omega, \tag{1}$$

where $\sigma^{\mathrm{T}} = \{\sigma_{xx}\ \sigma_{yy}\ \sigma_{zz}\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}\}$ is the stress vector, $\mathbf{b}^{\mathrm{T}} = \{b_x\ b_y\ b_z\}$ denotes the body force vector, $\mathbf{L}$ is the differential operator that can be written as

$$\mathbf{L}^{\mathrm{T}} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}. \tag{2}$$

The constitutive equation and the relationship between strain and displacement are given by

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}, \tag{3}$$

$$\boldsymbol{\varepsilon} = \mathbf{L}\mathbf{u}, \tag{4}$$

where $\mathbf{D}$ is the constitutive matrix. For the homogeneous materials, it can be expressed in terms of Young's modulus E and Poisson ratio ν as

$$\mathbf{D} = \frac{E}{(1+\nu)(1-2\nu)} \times \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2} & 0 & 0 \\ & \textit{symmetry} & & & \frac{1-2\nu}{2} & 0 \\ & & & & & \frac{1-2\nu}{2} \end{bmatrix}. \tag{5}$$

Boundary conditions of the problem domain Ω are given as follows

$$\mathbf{u} = \bar{\mathbf{u}} \quad \textit{on the essential boundary } \Gamma_u, \tag{6}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \textit{on the natural boundary } \Gamma_t, \tag{7}$$

where $\bar{\mathbf{u}}$ and $\bar{\mathbf{t}}$ are the prescribed displacement on the essential boundaries and traction on the natural boundaries, respectively, $\mathbf{n}$ is the vector of unit outward normal.

3. The idea and formulation of the presented algorithm

In this section, the gradient weighted formulation is described in detail. The problem domain is discretized into tetrahedral (or triangular for 2D space) elements for 3D space, as in the standard FEM. Upon the gradient weighted operation, the gradient field is formed by taking the element itself and its adjacent elements sharing common edges/faces into account.

3.1. Fundamental of gradient weighted operation

We briefly introduce the Shepard interpolation method/SIT [46,47] in this subsection. Given a problem domain Ω with the function $F(\boldsymbol{x})$, we decompose the problem domain Ω into n non-overlapping patches Ω_i, satisfying $\cup_{i=1}^{n}\Omega_i = \Omega$ and $\Omega_i \cap \Omega_j = \Phi\,(i \neq j, i, j = 1, 2, \ldots, n)$. The gradients of each associated patches are defined as $f_i\ (i = 1, 2, \ldots, n)$, then the interpolation scheme proposed by Shepard can be expressed as

$$F(\mathbf{x}) = \sum_{i=1}^{n} w_i(\mathbf{x}) f_i, \tag{8}$$

where the normalized weight function $w_i(\mathbf{x})$ has the following form

$$w_i(\mathbf{x}) = \frac{D_i(\mathbf{x})}{\sum_j^n D_j(\mathbf{x})}, \tag{9}$$

$D_i(\mathbf{x})$, a singular function at coordinate $\mathbf{x}$, can be expressed as

$$D_i(\mathbf{x}) = \alpha_i \varphi_i, \tag{10}$$

in which, α_i represents the weighting coefficient, φ_i represents the area for 2D space and volume for 3D space of patch i.

3.2. Reconstruction of the strain field using the gradient-weighted technique

As the problem domain is first discretized into a set of tetrahedron elements, the field variables $\mathbf{u}$ within each element are interpolated using the nodal displacements at the nodes of element through the linear shape functions in the following form

$$\mathbf{u} = \sum_{i=1}^{m} \boldsymbol{\Phi}_i(\mathbf{x})\,\mathbf{d}_i = \boldsymbol{\Phi}\mathbf{d}_e, \tag{11}$$

in which, $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_1\ \ \boldsymbol{\Phi}_2\ \ \boldsymbol{\Phi}_3\ \ \boldsymbol{\Phi}_4]$ is the vector of element shape function, $\mathbf{d}_e^{\mathrm{T}} = [\mathbf{d}_1\ \ \mathbf{d}_2\ \ \mathbf{d}_3\ \ \mathbf{d}_4]$ represents element displacement vector, m is the number of nodes of each element.

Substituting Eq. (11) into Eq. (4), the strain of each independent element can be obtained as follows

$$\varepsilon^e = \sum_{i=1}^{m} \mathbf{B}_i^e \mathbf{d}_i, \tag{12}$$

with

$$\mathbf{B}_i^e = \left[\Phi_{i'x} \quad \Phi_{i'y}\right]^{\mathrm{T}}, \text{ for 2D space} \tag{13a}$$

$$\mathbf{B}_i^e = \left[\Phi_{i'x} \quad \Phi_{i'y} \quad \Phi_{i'z}\right]^{\mathrm{T}}, \text{ for 3D space} \tag{13b}$$

For GW-FEM, both element itself and its adjacent elements are used to construct the strain field of each independent element with the help of Shepard interpolation function. Fig. 1 shows the schematic of part of a typical operation field. For the sake of simplicity, we define the domain (Ω^e_{ABCD}) as the master element, and its neighboring elements (Ω^e_{ABDk1}, Ω^e_{ABCk2}, Ω^e_{BDCk3}, Ω^e_{ACDk4}) sharing common faces are defined as adjacent elements. All these elements that are identically expressed as $\Omega_i^e\ (i = 1 \sim 5)$ compose a compacted supported domain.

In this approach, the gradient weighted operation is applied to the compacted supported domain on the strain vector ε. Based on the SIT mentioned in Section 3.1, the weighted strain vector of the eth element can be expressed as

$$\tilde{\varepsilon}^e = \sum_{j=1}^{n} \left[\omega_j(u) \cdot \varepsilon_j^e\right], \tag{14}$$

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